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This study utilizes Positioning Theory as a lens to analyze interactions between a teacher and her students. Using those interactions, this study seeks to better catalog and understand pervasive storylines in one teacher's secondary mathematics classroom as well as the intertwined positions of teacher and students within those storylines.

Additionally, this study amplifies the voice and lens of a teacher participant to showcase the perceived relationship between her reflexive and interactive positioning of herself and students during episodes of interaction. This single case study investigates one teacher's classroom practice over four years as she engaged in professional development and learning around high-quality, core instructional practices for teaching mathematics. Video recordings of classroom lessons and video-stimulated recall interviews were analyzed to illuminate referenced storylines about the meanings made of teaching and learning mathematics in this space and the positions assumed and afforded within.

This single case study provides unique insight into the evolution and evolvment of storylines and positions over time for this particular teacher while also honoring the relational and negotiable nature of positioning. Findings supported storyline development along three trajectories including those storylines and positions that remained consistent, others that dissipated, and still others that emerged over time. Additionally, findings suggest that professional development focused on pedagogical practice and student-centered instruction may support teachers in assuming more subdued, less powerful positions during classroom interactions and thus, affording

students more agentic, authoritative, and sense-making positions throughout inquiry driven mathematics lessons. Finally, findings suggest that as teachers shift to consider their assumed positioning in interactions, they have the ability to suggest, offer, and restrict particular positions for students. Implications for practice and research are discussed for teachers, teacher educators, professional development facilitators, and researchers.

THE CASE OF JAMIE: EXAMINING STORYLINES AND POSITIONS OVER TIME
IN A SECONDARY MATHEMATICS CLASSROOM

by

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CHAPTER I

INTRODUCTION

Many students are struggling in the United States to grow and learn mathematics. Unfortunately, many students are mathematically stagnant and underperform on standardized tests of achievement (Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006; Sztajn, Anthony, Chae, Erbas, Hembree, Keum et al., 2004). There are many test-related factors that may contribute to such results, including test validity, language, and even sampling procedures (Andrews, Ryve, Hemmi, & Sayer, 2014; Holliday & Holliday, 2003), but if instead of focusing on the measure, we take a closer look within mathematics classrooms, we would find that instruction is often teacher directed, teacher centered, and instructionally focused on procedural knowledge and processes (Stigler & Hiebert, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003). Often in such classrooms, student ownership and voice are limited (Cobb & Hodge, 2007), teachers as seen as the mathematical authority (Wagner & Herbel-Eisenmann, 2014), and students are often left positioned as “anti-intellectual” (Martin, 2009; Steele 2003) or stuck within societal racialized and gendered stereotypes (Bartell, 2011; Esmonde, 2011; Tholander & Aronsson, 2003). As a result of teacher directed models of instruction, students’ conceptual understandings remain underdeveloped and mathematical connections remain elusive and uninvestigated.

Schooling, especially in secondary mathematics classrooms, remains a space centered on teachers. Teachers make decisions about content, how to organize their instruction (i.e., lesson plans and timing), and often deliver lessons to students focused on procedures. This model limits students' agency, stifles their opportunities to make meaningful mathematics connections, and further alleviates opportunities of justification and argumentation that build and support mathematical proficiency (Stigler & Hiebert, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003). Teaching mathematics in such a way affords teachers roles of authority and holders of mathematical knowledge. Consequently, students are offered roles less authoritative, such as mathematical followers with limited sense-making opportunities and limited agency, power, and ownership of their learning. Such limited roles are particularly problematic when we consider the students in this classroom structure experiencing success and, more importantly, those students denied success and access. Marginalized and under-represented youth are often the ones denied experiences of mathematical success (Martin, 2009; Steele, 2003). So, students are left to question who is math for, who is it not for, and whom does it serve? Many students, including traditionally marginalized students, come to believe that mathematics is not for them. Teachers and students continually reiterate and weave exclusionary narratives about who can know mathematics while unintentionally denying particular groups of students' access and success in an imperative and gate-keeping content area.

Educational research in mathematics suggests a different model of teaching and provides evidence of its effectiveness for students (Boaler & Staples, 2008; Franke,

Webb, Chan, Ing, Battey, 2009; Fennema Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Stigler & Hiebert, 2004). High-quality mathematics instruction promotes conceptual understanding and mathematical proficiency using cognitively demanding tasks to engage students in inquiry, exploration, sense-making, and justification. In this model, students work collaboratively, building from another's mathematical thinking to co-construct deep, conceptual understandings. Students and teacher collectively facilitate small group and whole group discussions, sharing mathematical knowledge, power, and authority. During discussions, students are privy to a variety of applicable strategies as class members make connections to approaches shared by different groups. In this model, a teacher's instructional practice creates roles for students and him/herself not common in direct instruction. Agentic roles of power and sense-making are shared and collaboratively constructed rather than a teacher only assuming powerful roles. Traditionally marginalized students who may ascribe to a "math is not for me" mentality are afforded roles of power while experiencing mathematical success. As traditional/cultural narratives about whom math is for and whom mathematics serves are contested and disrupted, broader notions of mathematical success are constructed.

Enacting high-quality, equitable mathematics instruction is challenging.

Recently, efforts in teacher preparation have focused on decomposing the practice of teaching into core, learnable practices (McDonald, Kazemi, & Kavanagh, 2013; NCTM 2014; Teachingworks, 2016). Core practices have been defined as those that occur frequently in the work of teaching, are based on research, impact student learning, and maintain the complex nature of teaching (Forzani, 2014). Research in the field has shown

that classrooms incorporating core practices and opportunities for argumentation and reasoning discussions support and positively impact student learning (Boaler & Staples, 2008; Franke, Webb, Chan, Ing, Battey, 2009; Fennema Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Stigler & Hiebert, 2004) and have the potential to address issues of equitable instruction for traditionally marginalized students (Cohen, Lotan, Scarloss, & Arellano, 1999; Myers, 2014). High-quality instruction positions teachers and students in ways not typically available in traditional classroom structures. Narratives about who can do, learn, and be successful in mathematics, are broadened to be more inclusive and made available for a wider demographic of students.

Statement of Research Problem

Research has shown that traditional mathematics teaching practices limit opportunities for students to enact mathematical practices of justification, reasoning, and argumentation (Stigler & Hiebert, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003). Within classroom spaces, teachers are often seen as a source of mathematical authority (Sheets, 2005; Wagner & Herbel-Eisenmann, 2014) where teachers tell students ways to participate and engage with mathematics. However, high-quality mathematics instruction with an intentional lens toward equitable practice has the potential to provide a wider scope of narratives and roles to promote student voice, power, and authority, as more diverse groups of students experience success. In this dissertation, I investigate the meanings of teaching and learning mathematics in the classroom of a teacher engaged in a multiyear professional development project focused on high quality mathematics instructional practices. Specifically, I use Positioning Theory as an analytical and

theoretical lens to examine prevalent storylines and positions in one secondary mathematics teacher's classroom. I propose that through making often-implicit storylines and positions of a teacher and student explicit, mathematics teachers and mathematics teacher educators may collaboratively work to re-consider their reflexive positioning in classroom spaces. Through intentional self-positioning, a teacher may make broadened, more agentic positions accessible for students. This study investigates the meanings that one teacher and her students make of teaching and learning mathematics when instruction is approached in an open-ended, student-centered, conceptually rich manner and the ways those meanings position a teacher and students in the classroom.

Overview of the Dissertation

This dissertation is organized into five chapters. Chapter I introduces the research problem and highlights its significance within mathematics education research. Chapter II provides a review of relevant literature, as well as describes the theoretical and analytical framing used throughout the study. Chapter III briefly explains the context of the study and provides a description and rationale for case study methodological research, the approach used to answer the aforementioned research questions. I also provide data sources and analytical measures, address issues of validity, and investigate researcher positionality. In Chapter IV, Positioning Theory is used as an analytical lens to understand the meaning of teaching and learning mathematics storylines, the assumed and afforded positions within such storylines, and the ways those change over time. I further address the teacher's perception and interpretation of storylines and the nature of

positioning through video-stimulated recall interviews. Finally, Chapter V concludes with recommendations for educators, professional development facilitators, teacher educators, and educational researchers.

CHAPTER II

REVIEW OF LITERATURE AND THEORETICAL PERSPECTIVE

In this chapter, I provide a review of the research literature pertinent to the scope of this study. I begin with a picture of secondary mathematics classrooms today, compare and contrast two common forms of instruction: dialogic and direct. Then I move to discuss the structure, norms, and common narratives of teaching and learning in each instructional space. Next, I introduce positioning theory as a potential entry point to begin to consider the work of narrative development and roles in more dialogic classroom environments. Using this theory, I conclude by stating the specific research questions guiding this study.

Mathematics Teaching Today

In the United States, mathematics teaching and learning at the secondary level is largely teacher-directed and focused on procedures (Stigler & Hiebert, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003). A typical teacher-centered and driven approach to teaching limits students' opportunities for mathematical justification, reasoning, collaborative discussion, and constructive argumentation; all of which are crucial in developing productive mathematical dispositions in students (Stigler & Hiebert, 2004). This approach to teaching is marked by classroom discourse and norms for interaction that follow a standard Initiate-Respond-Evaluate structure within these

classroom spaces which begins as a teacher poses a question, students respond, and then the teacher evaluates the response in some way (Mehan, 1979). While this instructional approach is productive and necessary in some instances, more often students are limited in their opportunities to mathematically make sense, conceptually understand, justify and create meaningful connections. Teacher-centered pedagogies and initiate-respond-evaluate approaches to classroom discourse tend to shape and reproduce unproductive meanings of teaching and learning mathematics and leads to inequitable opportunities for students success and access in mathematics.

Models of Mathematics Instruction

Within mathematics education, debates around two forms of instruction exist, and though they are often broadly referred to as “traditional” and “reform”, Munter, Stein, and Smith (2015) categorize instruction as direct and/or dialogic. Direct instruction is conceptualized as classrooms following an IRE (initiate-respond-evaluate) structure where students are taught a particular concept, lead through a series of guided and independent practice problems, and are provided evaluative type feedback (Clark, Kirschner, & Sweller, 2012; Mayer, 2004). Dialogic instruction, on the other hand, suggests more student engagement through productive struggle, mathematical justification, critiquing the reasoning of others, and carefully selected practice for students to engage with (Schoenfeld, 2002).

Though Munter et al. (2015) noted some similarities between these two models, such as both perspectives value the use of mathematics tasks and opportunities for independent practice; they also identified and described nine distinctions between the

models. These distinctions include the role of talk, the role of group work, sequencing of topics, the nature and ordering of tasks, teacher feedback, emphasis on student creativity, the purpose of diagnosing student thinking, the role of mathematical definitions, and finally, the role of representations. They posit that these distinctions were the result of differing underlying conceptions of what it means to know and learning mathematics:

Between these two models, perspectives on learning are even more distinct than those on knowing. The perspective underlying the direct instruction model is that, when students have the required prerequisite conceptual and procedural knowledge, they will learn from (a) watching clear, complete demonstrations—with accompanying explanations and accurate definitions—of how to solve problems; (b) practicing on similar problems sequenced according to difficulty; and (c) receiving immediate, corrective feedback. The perspective underlying the dialogic model, on the other hand, is that students must (a) actively engage in new mathematics, persevering through challenges as they attempt to solve novel problems; (b) participate in a discourse of conjecture, explanation, and argumentation; (c) engage in generalization and abstraction, developing efficient problem-solving strategies and relating their ideas to conventional procedures; and, to achieve fluency with these skills, (d) engage in some amount of practice (p. 13).

Though these models are theoretically incompatible, Munter et al. (2015) assert that the daily work of mathematics teaching likely incorporates elements of each model. While practices a teacher enacts during teaching a conceptually oriented lesson may be more closely aligned with a dialogic model, she may use a different set of practices from a more direct model. In reality, a teacher's model of instruction is most likely a hybrid of the two models. In fact, a practice can be a part of either model (e.g. using cognitively demanding tasks). Over time, experiences in these classroom spaces accumulate and communicate particular meanings of teaching and learning that are reproduced and

communicated through daily interactions. Thus, instruction and practices can be a part of both models, but result in different outcomes because of the underlying meanings of teaching and learning.

Narratives and Roles

Researchers have identified a variety of commonly held meanings of mathematics and made explicit the often-implicit narratives of what it means to do, to teach, and be successful in mathematics classrooms. Narratives are a “way of making sense of human actions and a way of knowing” that support individuals in “giving meanings to experiences” (Chapman, 2008). Narratives support individuals in understanding oneself and making sense of one’s life experiences (Chapman, 2008).

A number of studies in mathematics education have investigated narratives about students and learning. Some students have revealed narratives about students’ mathematical ability and tracking of students (Horn, 2007; Oakes, 1992; Stiff, Johnson, & Akos, 2011; Suh, Theakston-Musselman, Herbel-Eisenmann, & Steele, 2013). These narratives are evident in the ways teachers’ talk about what students can and cannot do (Herbel-Eisenmann, Johnson, Otten, Cirillo, & Steele, 2015; Wilson, Sztajn, Edgington, Webb, & Myers, 2017) and the ways students are institutionally tracked with little mobility across tracks (Suh et al., 2013). Some researchers have identified a maturation narrative that attributes mathematical ability to an age and/or a grade level and describes who is mature enough for particular types of mathematics (Suh et al., 2013; Thompson, Philipp, Thompson, & Boyd, 1994). Others describe narratives about the two distinct ways to teach mathematics alluded to earlier, including traditional, more procedural

methods versus student-centered, collaborative methods of teaching (Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, and Schaar, 2005; Herbel-Eisenmann, Sinclair, Chval, Clements, Civil, Pape, & Wilkerson, 2016; Klein, 2003; Munter, Stein, & Smith, 2015; Schoenfeld, 2004) with conceptions of more traditional methods being “telling” practices (Lobato, Clarke, & Ellis, 2005). Still other narratives in classrooms include mathematical answers representing evidence of knowledge and understanding (Tait-McCutcheon & Loveridge, 2016), teachers limiting positions for students based on their beliefs of what competence should look like (Davies & Hunt, 1994), female students acting as “subteachers,” a common narrative in elementary classrooms (Tholander & Aronsson, 2003) and other gendered narratives (Esmonde, 2011), students assuming expert and non-expert roles in group work based on cultural identities (Battey, 2013; Esmonde, 2009; Martin 2007) and finally, narratives situating teachers as both mathematical and classroom authorities (Sheets, 2005; Wagner & Herbel-Eisenmann, 2014).

Narratives such as those listed above are not unique to singular classrooms; rather, they are common and consistently reproduced in concert with particular meanings of teaching and learning mathematics. Not only do teachers and students convey these meanings of teaching and learning, but also individuals from school communities at large, university and research communities, and even societal norms. For example, many mathematics teachers and students, math departments, and other school officials would ascribe to the narrative of correctness as evidence of understanding (Tait-McCutcheon &

Loveridge, 2016), such that if a student produced a correct answer, most would attribute that to a certain level of knowledge, skill, and/or understanding.

In general, narratives assign particular roles to individuals. Roles are expected norms of behaviors for individuals that determine the rights, obligations, and modes of interaction. For example, teachers may assume more authoritative roles such as knowledge-holder under a narrative of direct modeling instruction. Narratives about the meaning of teaching and learning mathematics determine particular roles for students and teachers that assign certain expectations and limitations to individuals that may afford or constrain modes of communication for each individual. For example, within a narrative where only teachers possess mathematical knowledge and understanding, teachers assume roles of authority and knowledge-holder while students are limited to underprivileged, subservient roles and denied mathematically authoritative roles. Roles adopted by teachers and students determine what is socially acceptable and appropriate by establishing rights, responsibilities, obligations, and expectations for engagement. Depending on the roles assumed by teachers and afforded students, students may be granted or denied access to conversation and thus have either broadened or constrained means of participation.

Instruction more aligned with direct models of instruction may have evidence of narratives around precision, accuracy, quickness, organization, etc. Within such narratives, a teacher and students are assigned roles of mathematical authority and recipient of knowledge, respectively that reproduces particular meanings of teaching and learning mathematics centered on giving and receiving knowledge. For example, within

a direct instruction classroom, a narrative of teaching means correcting students, a teacher assumes a role of authority and a student may be complicit in following directions and steps through a procedure to arrive at an answer. Conversely, within more dialogic models of instruction, a classroom narrative may suggest that teaching is about sense-making, such that the teacher assumes a role of facilitator as she makes sense of what students are doing, and works to connect student ideas. If teaching means sense-making, then students are positioned as sense-makers as well and tasked with collaboratively reasoning, building on, and critiquing the work of their peers.

Further, research has shown that a student's role in classroom interactions matters because of the implications of roles on students' power and authority in the room as well as a student's agency and ownership of content (Cobb & Hodge, 2007; Turner, Dominguez, Madonado, & Empson, 2013). Once a student has been positioned or has assumed particular roles consistently over time, these repeated positions could cultivate students of certain "kinds" (Anderson, 2009) that position particular students or groups of students as "anti-intellectual" (Martin, 2009). Developing students as particular "kinds" or as occupying specific categories often results in institutional labels for students such as "gifted" or "learning disabled." positioning over time is derived from experiences in a learning space has implications for mathematical identity development for students and the ways students perceive and relate to mathematics moving forward (Anderson, 2009).

When teachers work to shift the focus of their instruction, they re-negotiate their role in the classroom and alternative, more privileged roles may become available to students that can counteract negative identity development. Models of instruction that

are intentional and explicit in the ways teachers and students are positioned during teaching and learning may support teachers in assuming different roles during interactions that suspend teacher authority and instead, give rise to students' power, agency, and mathematical authority such that more students see and enter mathematics communities (Cohen & Lotan, 1997) that were otherwise off limits and not viewed as inviting.

Summary

Mathematics teaching in secondary schools may follow two types of models: direct or dialogic. Models need not be viewed in isolation, as approaches to teaching are neither strictly direct nor dialogic, rather classroom practices are the same in each model, it is the meaning behind the practices that differ. The meanings made of teaching and learning in direct and dialogic instruction are significantly different and suggest different narratives and roles for teacher and students. Thus, each model of instruction has implications for the narratives and roles at work in classroom spaces.

Core Practices of High-Quality Mathematics Instruction

High-quality mathematics instruction is a form of dialogic instruction in which instructional practices are deemed high-quality across three dimensions including (1) students with opportunities to collaboratively engage in rich mathematics tasks through participation in (2) collective, co-constructed mathematics discussions as a (3) teacher facilitates discussion and learning (Munter, 2014). Though there are many characterizations of this type of instruction in the field including *high-quality* (Munter, 2014), *ambitious* (Forzani, 2014; Jackson & Cobb, 2010; Kazemi, Franke, & Lampert,

2009; Lampert, Franke, Kazemi, Ghouseini, Turrou, Beasley, et al., 2013), *adaptive* (Cooney, 1999; Daro, Mosher, & Corcoran, 2011), *complex* (Boaler & Staples, 2008), and *responsive* (Edwards, 2003; Jacobs & Empson, 2015), all share a primary goal of promoting mathematical sense-making and proficiency such that all students may be successful. Research has shown that this type of instruction leads to increased student learning and performance (Boaler & Staples, 2008; Franke et al., 2009; Stigler & Hiebert, 2004; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008), but identifying the instructional practices that comprise this model has proven more difficult.

Research on teaching has demonstrated that high-quality instructional practices are difficult for novices to learn and difficult to enact (Cohen & Ball, 1990; Edgington 2012; Myers, 2014). In recent years however, teacher educators and researchers have made progress in developing pedagogies of practice and have begun using Grossman and colleagues' (2009) idea of decomposing teaching into core, learnable practices. Such smaller practices allow teacher educators to support prospective and practicing teachers in learning to enact high-quality instruction in mathematics classrooms. Grossman et al. (2009) define core practices as practices that occur frequently in the work of teaching, are based on research, impact student and teacher learning, and also maintain the complex nature of teaching (Grossman, Hammerness, & McDonald, 2009).

In practice, the work of high-quality instruction supports students' development of mathematical proficiency by moving beyond students' procedural understanding to include reasoning, argumentation, and critique of others' reasoning. Teaching in such a way requires a skill set and establishing clear learning goals for students while also

maintaining learning environments that support student engagement, talk, and collaboration. Additionally, for students to have opportunities to critique the reasoning of others, teachers must foster and facilitate robust mathematical discussions, while developing norms for collective and equitable engagement. These mathematical discussions provide students with opportunities to justify their own mathematical reasoning, construct individual and collective meaning about important mathematics, and provide teachers with useful knowledge of the ways in which students are making sense of their mathematical work in relation to the work of others and the mathematical goal of the lesson (Staples & Colonis, 2007). Research in the field has shown that classrooms incorporating opportunities for argumentation and reasoning discussions have been shown to support and positively impact student learning (Boaler & Staples, 2008; Fennema Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Franke, Webb, Chan, Ing, Battey, 2009; Stigler & Hiebert, 2004) and may also have the potential to address issues of equitable instruction for traditionally marginalized students (Myers, 2014). Exactly what instruction that supports students in these ways looks like in a classroom setting is deeply engrained in the individuals and context, which is one reason the field has struggled to identify an agreed upon set of core and learnable mathematical instructional practices.

While the idea of core practices has been taken up and serves as a focus for research and collaborative development in teacher education (Core Practices Consortium, 2018), identifying and agreeing upon a shared set remains a significant challenge. In their recent review of research on mathematics teaching, Jacobs and Spangler (2017) noted

four tensions inherent in the quest to identify core practices of teaching including (1) determining criteria for core practices, (2) identifying sets of core practices to target, (3) using common language, and (4) attention to the relational nature of core practices. They ended by encouraging the field to use work with core practices as a way to disrupt power dynamics in mathematics, elevate the voices of teachers, and deeply ground practices in the unique context and community of the classroom space.

Hence, rather than foreground particular practices, I want to consider the ways practices position students and teacher during classroom interactions. This study investigates the meaning of teaching and learning mathematics for one teacher engaged in sustained professional development around student-centered, high-quality mathematics instructional practices to understand the meanings of teaching and learning mathematics in her classroom, narratives present in the classroom and roles assumed by the teacher and assigned to students. Research has shown that repeated positioning impacts students identity development (Anderson, 2009; Suh, Theakston-Musselman, Herbel-Eisenmann, & Steele, 2013) and as a subject, mathematics is often a gate-keeper (Martin, Gholson, & Leonard, 2010; Stinson, 2004), denying access for students' continued progression in particular fields of study. To better understand the meanings of teaching and learning mathematics, roles assigned by teacher and students in a classroom space, and the ways instructional practices position students, I use positioning theory as a lens. Positioning theory allows one to analyze episodes of interaction using spoken and non-verbal cues between teacher and students to consider meanings made and positions assumed.

In the next section, I introduce positioning theory and describe three constructs within the theory. I discuss ways positioning theory has been used in the field, as well as the ways I use the theory that is similar and different to others use. Finally, I address the co-defined nature of positioning and other relational aspects to consider when using the theory to analyze classroom interactions.

Theoretical Perspective: Positioning Theory

Positioning theory is the study of moment-to-moment interactions when people are positioned and located in particular ways within an episode of interaction with an implied and understood storyline under which these interactions and positions take place. Participants' positions grant or restrict certain "rights and obligations of speaking and acting" (Harré & van Langenhove, 1999, p. 1). Some may conflate the meanings of role and position, but positioning theory sees these terms as distinct. Whereas positions are relational, fluid, and contestable, roles are conceptualized as static and restrictive (Harré & Slocum, 2003). Though positions are fluid and shift moment-to-moment, some have started to think about repeated positioning over time and the impact of such repeated positions on static roles and storyline meanings. Anderson (2009) speaks to repeated positioning as developing students as certain "kinds" and highlights the implications of these experiences on identity development. Rather than focus on the effects of repeated positioning in terms of identity work, this study focused instead on the impact of repeated positioning on the meanings made of teaching and learning mathematics and how meanings impact storyline development in a classroom. Thus, rather than think about the ways storylines impact positions and communication acts, I approach this work to

understand how repeated positions have the potential to impact the storylines and meanings made of teaching and learning mathematics in one classroom.

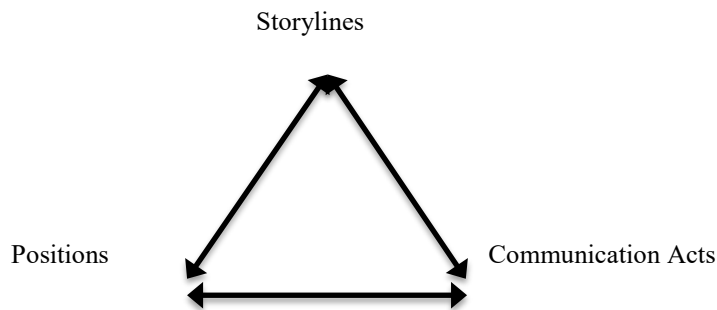


Figure 1. Positioning Triad

Positioning theory is conceptualized using episodes of interaction around a particular idea or thought. Episodes are theoretically structured by three mutually constitutive constructs and visually represented using a positioning triad (Figure 1). Each vertex represents a construct and each are connected with a double-headed arrow to indicate the mutual and dependent relationship that exists between each. At the top of the triangle are socially constructed narratives referred to as *storylines*. Storylines are “broad, culturally shared narratives that act as the backdrop of the enacted positionings” (Herbel-Eisenmann et al., 2016). The storyline not only determines the types of positions available, but also the ways positions are enacted. The second construct is the explicit and implicit speech acts and non-verbal cues that participants say and do within an episode of interaction. Those utterances and non-verbal cues together create a participant’s *communication acts* (Herbel-Eisenmann, Wagner, Johnson, Suh, and Figueras, 2015). Finally, the third construct is the *position(s)* each participant is assigned or assumes, and

each position is bounded by rules and obligations of interaction. Positioning can be explicit but is more often implicit and unintentional. Positions are continually accepted, contested, and negotiated between participants throughout interactions (Harré & van Langenhove, 1999).

Though storylines and communication acts serve as the foundation for understanding positions, it is imperative to understand storylines, communication acts, and positions as a collective, interactive, and mutually determining unit with each construct co-defining the other (Harré & Slocum, 2003; Harré & van Langenhove, 1999; Herbel-Eisenmann et al., 2016). The interrelated nature of all three constructs means that as a storyline shifts, so too will positions and ways of speaking and acting (Herbel-Eisenmann et al., 2015). To be clear, however, this is not to say that only storyline shifts initiate change; rather, shifts are initiated by all three constructs working together simultaneously.

It is important to note that due to the relational nature of positioning, any interaction is an act of positioning. Within episodes of interaction, participants are necessarily positioned, but not all acts of positioning are intentional. Reflexive positioning refers to the intentional or unintentional positioning of oneself (Harré & van Langenhove, 1999). Similarly, positioning another in an episode refers to interactive positioning but may be contested and negotiated by participants within the interaction. To illustrate, consider a teacher in a classroom and leading a culminating discussion after students have investigated a particular mathematical idea. During this discussion, the teacher intentionally positions herself as an inquirer seeking to understand what students

have done and ask questions to make connections between student ideas. As a teacher assumes this position, her verbal and non-verbal cues may encourage, support, and even deter students in explaining their thinking, reasoning, and justification of ideas. Thus, students are often interactively positioned as having little mathematical authority and rarely as agentic mathematical sense-makers. This reflexive and interactive positioning may draw from a storyline that mathematics is about teacher authority and only for certain students. Other times, students are mathematical sense-makers in the classroom, supported by communication acts as the teachers sits in a student desk in the back of the room and pushes students to display their work, ask questions of one another, and justify their thinking. The physical location of teacher and students is a type of non-verbal communication act that visually reiterates positioning and weaves narratives around the meaning of teaching and learning mathematics.

Since all positioning is negotiable and relational, a student's positioning is in relation to a teacher's positioning and vice versa. Those positions may be accepted or rejected by individuals in the interaction. Students could accept less powerful positions and allow a teacher to authoritatively lead the class through their mathematical thinking process. Likewise, students may contest their positioning of mathematical authority by giving short responses and not wanting to elaborate on their thinking. Ultimately, the positioning of the teacher and student in such an episode determines access to and choices in discourse. Less powerful positions are limited in the ways they may engage in content and repeated positions and narratives around the meanings of teaching and learning impact future positions made available moving forward in this classroom and

even beyond this classroom space. Thus, reproduced meanings of teaching and learning and repeated positions of teacher and student become norms of interaction and impact a student's mathematical disposition, identity, and agency moving forward in their mathematical careers.

Others in the field have used positioning theory to consider sources of authority in mathematics classrooms including the authority of individuals, processes or actions, classroom objects, and disciplinary artifacts (Wagner & Herbel-Eisenmann, 2014), to understand teacher identity in literacy classrooms (Hall, Johnson, Juzwik, Wortham, & Mosley, 2010), students' participation and positions in group work activities (Tholander & Aronsson, 2003), student identities (Anderson, 2009), classroom discourse (Kayi-Aydar & Miller, 2018), as well as issues around equity and access for students (Tait-McCutcheon & Loveridge, 2016). This study is similar in that the theory is used as a lens through which to analyze classroom practice, instruction, and positioning. What is unique about this study, and where this study contributes to the growing body of literature around positioning is the prominence and amplification of teacher voice and reflection. Not only will I use positioning theory as a researcher, but the theory will also serve as a lens and a tool for a teacher to reflect on practice and positioning in her classroom over three years while engaged in professional development.

Summary

Using positioning theory as a theoretical and analytical lens, this study seeks to better understand prevailing storylines within the classroom context around the meaning of teaching and learning mathematics. The intertwined and negotiated nature of

positioning means that assumed and afforded positions within storylines about mathematics teaching and learning are just as necessary to understanding the phenomenon. Thus, I use all constructs of positioning to examine the meanings of teaching and learning in the classroom of a teacher participating in a multiyear professional development project focused on core practices of high quality mathematics education. Specifically, I use positioning theory to refine my broad research question in the following ways:

Research Question 1: In what ways do storylines and positions change as a teacher engages in sustained professional development around instructional change?

Research Question 2: How does a teacher perceive the relationship between interactive and reflexive positioning?

CHAPTER III

METHODOLOGY

To explore prevalent storylines and positioning in a teacher's mathematics classroom, I used case study methodology. Before introducing the case, I first discuss the professional development program that served as context for this study, including its structure, goals, and evaluation. Next, I describe case study design, provide background and justification for the use of case study methodology to address my research questions, I describe the selection criteria and uniqueness of this case, as well as concerns related to validity. After describing analysis methods, I conclude with issues of researcher positionality.

Context

Research has shown that effective professional development (PD) opportunities can be characterized as intensive, marked with opportunities for authentic engagement and experience (Ball & Cohen, 1999; Wilson & Berne, 1999), promoting transferability into classroom practice (Goldsmith, Doerr, & Lewis, 2014), and finally, in order for this to be attained, PD must be ongoing and sustained over time. Project CMaPSS (Core Mathematics Instructional Practices in Secondary Schools) was a partnership between a university and a neighboring school district, funded by several sources, including United States Department of Education, North Carolina Department of Public Instruction, and the mathematics education group at the Universities' School of Education. The

partnership worked to maintain key objectives that addressed needs of the school district as well as the needs of individual teachers. This PD began spring 2015 and lasted three years, extending through the 2015-2016, 2016-2017, and 2017-2018 school years.

This PD focused on core instructional practices (McDonald, Kazemi, & Kavanagh, 2013; NCTM 2014; Teachingworks, 2016). It was grounded in classroom context, responsive to teachers' goals for their instructional practice, and aligned with proposed district initiatives. Each year of the PD was marked by a summer and school year component. Each summer, teachers worked for two weeks (60 hours) in a Summer Institute to learn mathematical content, build mathematical knowledge for teaching, and support each other in learning and enacting core instructional practices. Each Summer Institute was organized around cycles of investigating core practices by engaging with representations, decomposing practice, and approximating core practices of high quality mathematics instruction during rehearsals.

While specific goals shifted each year to meet the needs of the district and teachers, PD activities remained consistent. Teachers participated in a variety of activities during Summer Institutes including rehearsals, engaging in math tasks as learners, developing student-centered lessons, observing other teachers' instruction, and reflection. Rehearsals (Lampert et al., 2013) were structured in ways that allowed each teacher to "rehearse" a particular practice or move. As professional development facilitators, our team focused on larger practices within a lesson including *launching* a task (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013), *monitoring* students as they engage (Kazemi & Hubbard, 2008), and finally, leading whole class *discussion* (Stein,

Engle, Smith, & Hughes, 2008). While some teachers acted as students, other teachers posed reflective questions and then together all teachers reflected and provided feedback on the rehearsal (Webb, 2018).

A primary goal of Project CMaPSS was for participating teachers and coaches to learn and enact core practices of high-quality mathematics instruction that make use of research on students' mathematical thinking in their classrooms. In order to meet this goal, members of the research team examined research focused on core practices specifically, and mathematics teaching generally, and developed a conceptual framework for instructional practice (see Webb, 2018) around segments of a task-based lesson. Those segments including *launching* an instructional task, *monitoring* student engagement with the task, and finally bringing student work and reasoning together to build a collective *discussion* with all students.

Our team also worked to identify teacher discourse moves as smaller grain-sized practices that would support students' engagement with the task. Our working assertion in working with teachers on moves and goals for those moves was that moves were perhaps the appropriate grain size to enter the work of learning and enacting high-quality instruction with teachers. So, in addition to larger practices of *launching*, *monitoring*, and *discussing*, we decomposed practices into a set of teaching moves and focused on the reasons for making particular moves. We chose moves that could be nested within and used across each of the larger practices throughout a lesson. Herbel-Eisenmann, Steele, and Cirillo (2013) identified six teacher discourse moves including waiting, inviting a student to participate, revoicing, asking a student to revoice, probing, and creating

opportunities for students to make sense of another's work. Our team used these teacher discourse moves to generate a list of moves we felt held across all interactive core practices. The five moves we selected were *probing* students' thinking, *revoicing* a student's contribution (Chapin & Anderson, 2013), *explaining*¹ a mathematical idea (Lobato et al., 2005), *pressing* students' reasoning (Kazemi & Stipek, 2017), and *orienting* students to one another's ideas (Kazemi & Cunard, 2016). Table 1 below represents moves underscored during our professional development along with a brief description of each move. During the school year, teachers and facilitators met at least twice during the fall semester and twice during the spring semester to focus on particular pedagogical problems of practice, mathematics content, or personal reflection through video clubs (Sherin & van Es, 2005).

Though a complete description of this professional development program and its context is beyond the scope of this paper (see Webb, 2018), it is important to emphasize the foundation and professional learning context for the case, Jamie. This professional development has been evaluated each year and has been shown to be effective each year (Duggan & Jacobs, 2016, 2017). Thus, rather than taking an additional evaluative approach to the PD, this study focused on the process of changing and redefining instruction for the purpose of informing future PD design. Specifically, given this professional development context, this study aimed to better understand the changes in

¹ Our use of the practice of explaining is equivalent to Lobato et al.'s (2005) notion of telling. Given its connotation and widespread misinterpretations that student-centered instruction means "don't tell," we chose to use the term explaining rather than telling to refer to this practice in the professional development.

meanings of mathematics teaching and learning in Jamie’s classroom as she her teaching became increasingly aligned with high quality mathematics instruction.

Table 1

Instructional Moves

Move	Description
Revoicing	Repeating or rephrasing a student’s previous contribution related to the task, representation of the task, or students’ mathematical thinking.
Probing	Asking a question to seek information students have verbalized or recorded.
Pressing	Asking a question or making a statement to encourage students to explain or justify their reasoning beyond what has already been evidenced or to extend their thinking to a new or related idea.
Orienting	Asking a question or making a statement that encourages students to hear, use, or connect another idea to their own idea.
Explaining	Making a statement to clarify the task, representation, or approach.

Design

Case Study Methodology

Case study is a methodological form of qualitative research that provides researchers opportunities to gain unique perspective and understanding of a “bounded system” or case (Creswell, 2013). Teaching and learning are socially and culturally situated and complex to understand; additionally, positioning offers a relatively new lens on classroom interactions between student and teacher. Case study is generally selected

to understand and make sense of particular issues in practice tied tightly to context and is often exploratory in nature (Creswell, 2013; Yin, 2013). For these reasons, case study represented an appropriate methodological approach. This study sought to understand the case of Jamie, a secondary mathematics teacher who represented a bounded case within her secondary mathematics classroom context. To understand constructs of positionings and storyline development within the case, this study used intrinsic case study methods to explore Jamie's practice (Stake, 1995) and longitudinal analysis over the course of three years to understand and explore Jamie's individuality as a secondary mathematics teacher, highlight unique and pervasive storylines within her classroom, as well as the assumed and afforded positions (Creswell, 2013).

Case study researchers work to understand deeply the uniqueness of a particular case and the complex, interconnected nature of the case with its context (Creswell, 2013; Stake, 1995; Yin, 2013). While this deep understanding of one case may not support researchers in making generalizations, grand or otherwise, this methodology does promote particularization of the case itself—understanding well the particular aspects, inner workings, and distinctiveness of the case (Stake, 1995). For that reason, this study did not make generalizations beyond the existence of storylines and positions present in Jamie's classroom to secondary mathematics classrooms, but rather revealed Jamie's unique contextual elements that supported and/or constrained storyline development and evolution within her classroom, as well as the afforded and assumed positions granted within those storylines. By understanding the particular and unique nature of Jamie, the field is granted rare insight in the *evolution* of positioning and storyline development, as

well as Jamie's perception of positioning in her classroom that may otherwise go unnoticed.

I chose a single, exploratory case study to analyze the research questions in this study to make sense of Jamie, a teacher in a secondary mathematics classroom. Reflexive and interactive positioning and storyline development are complex phenomena; thus by narrowing the focus to illuminate the case of Jamie and her classroom, this study explored the personal and unique nature of Jamie's reflexive and interactive positioning, storyline progression in her classroom over time, and Jamie's perception of the related nature of positioning.

Case Selection

Jamie was part of a larger research study investigating teacher learning of core mathematics instructional practices and worked with this project for three years. Jamie represented a unique case, as she was one of only two middle grades educators in a PD predominantly attended by high school mathematics educators. While participation in the PD varied from year to year, there were at least 12 and at most 18 teachers and teacher leader participants each year. Jamie taught eighth grade mathematics and Math 1 in a yearlong format. She has been an educator for 16 years and had spent the last 12 years working at her current school site.

Jamie's classroom was purposefully chosen (Maxwell, 2013) primarily for two selection criteria. First, the accessibility of Jamie's classroom and the productive relationship fostered with Jamie helped the researcher to better understand and make sense of the research questions (Yin, 2013). Additionally, Jamie represented an atypical

and unique case due to her yearlong course format. Other teachers in the professional development had a semester-based schedule; hence, Jamie's classroom provided a consistent student population in fall and spring observations. This was a significant selection criterion because storylines and positions unique to Jamie may or may not be directly linked to the students in the room. Hence, keeping a stable student cohort for fall and spring observations provided an added layer of consistency.

This study was conducted in a small, rural school district in the south. The school district was composed of four traditional high schools, one alternative high school, four middle schools, and 15 elementary schools. All five high schools and one middle school were represented in the teacher and teacher leader population that attended the PD. This study was conducted at one of the local middle schools that served roughly 800 students in Grades 6 through 8. This school site served a student population identified as 78.8% White, 9.4% African American, 7.5% Hispanic, and 4.3% other. At the time, roughly 38% of students in this school received free and reduced lunch.

Though Jamie taught both eighth grade mathematics and Math 1, for the purposes of this study, as well as in alignment with goals of the professional development, video recordings were only conducted in her Math 1 or high school leveled courses. Because these students were tracked into Math 1 in eighth grade, the class demographics were not as diverse and were not aligned with enrollment percentages for the school. Jamie's classroom comprised a student population of predominantly white, middle-class students labeled as strong, moderate, or individual AIG (Academically and Intellectually Gifted).

Design of the Study

Though formal design is rare in case study research, Yin (2013) encourages researchers to consider formal design to promote both rigor and quality. Following this suggestion, I created and utilized a formal, two-phase design to investigate the evolution of storylines and positions in this space. Further, case study researchers are encouraged to both define and bound the case when conducting case study research (Yin, 2013; Creswell, 2013). In accordance, I developed an embedded, single-case design with one unit of analysis (Yin, 2013; Creswell, 2013; Stake, 1995).

This study examined the case of Jamie's classroom with Jamie as the single, embedded unit of analysis. Jamie's case was bounded by the classroom context and course, as well as being bounded chronologically over the span of three years of project work. A diagram of the study design is depicted in Figure 1. Utilizing one unit of analysis, I focused on Jamie as a single teacher, embedded within the case of her classroom and context of the middle school community in which she worked. Additionally, I analyzed the potential shifts and evolution in storylines and afforded positions over time as the teacher engaged in sustained professional development, and worked to understand Jamie's perception of teacher and student positioning.

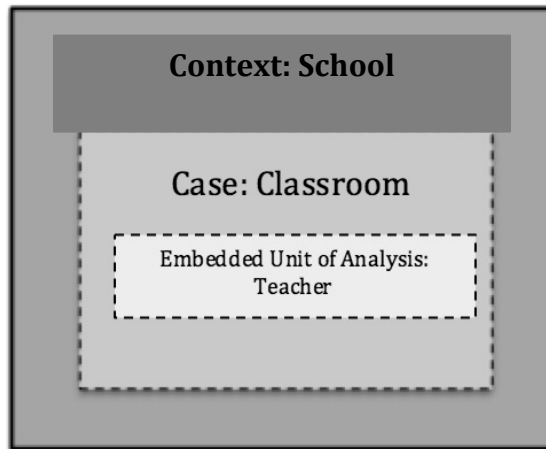


Figure 2. Case Study Design

Data Sources and Analysis

In what follows, I describe sources of data and the timeframe of the data collection, align data sources with research questions, and detail the data analysis procedures for each data source. Next, I address issues of validity by describing my time spent in the field, triangulation of findings from various data sources, member-checking measures with Jamie, and consideration of alternate conclusions. Finally, I conclude by examining myself and speak to my own researcher positionality and my potential impact on the study.

Table 2

Data Sources and Timeline

		Timeline					
Data Collected	Phase	Spring 2015	Fall 2015	Spring 2016	Fall 2016	Spring 2017	Spring 2018
Field Notes	1 & 2	X	X	X	X	X	
Video Observation	1	X	X	X	X	X	
Video-Stimulated Recall Interviews	2						X

Sources

Data consisted of recorded field observations of classroom lessons, transcribed video lessons, interviews with Jamie after a particular classroom lesson, and video-stimulated recall interviews. During phase one, classroom lessons were recorded once in the fall and once in the spring, after approximately two months of instruction, across three school years of work with the teacher. Each observation was also transcribed for research purposes. In phase two, I conducted semi-structured interviews with Jamie using a video-stimulated recall process to highlight Jamie's thoughts and musings about particular interactions. These video-stimulated recall interviews supported member-checking and were audio-recorded, transcribed, and field notes were collected. Data analysis and collection ran somewhat concurrently so that I could make decisions regarding new and pertinent data collection needed for the study (Miles, Huberman, &

Saldaña, 2014). Table 2 aligns data sources with a time frame of collection and table 3 uses data sources collected to support each research question addressed in the study.

Table 3

Data Sources and Research Questions Matrix

	Field Notes	Video Observation	Video- Stimulated Recall Interviews
Research Question 1 <i>In what ways do storylines and positions change as a teacher engages in sustained professional development around instructional change?</i>	X	X	X
Research Question 2 <i>How does a teacher perceive the relation between interactive and reflexive positioning?</i>			X

Data Analysis Procedures

This study utilized an inductive (Creswell, 2013; Miles, Huberman, & Saldaña, 2014; Yin, 2013) analysis process. Recordings of each classroom lesson were initially viewed while I wrote tentative memos and ideas about what was transpiring in the classroom. Next, video transcripts were divided into interactive teaching practices (launching, monitoring, leading discussions) and open coding (Strauss & Corbin, 1998) was used to break-down, examine, and identify relevant themes in recordings about the

nature of teaching and learning mathematics in the classroom space. Each practice was further dissected into episodes of interaction.

Episodes of Interaction. Perhaps one of the most difficult tasks during analysis was determining what counted and was evidence of an episode of interaction. Harré and van Lagenhove (1999) describe an episode as a “sequence of happenings” where persons in an interaction engage in dialogue and include the behind the scenes ideas of “thoughts, feelings, intentions, and plans” (p. 5). An episode is not only characterized by how participants engage, but also determines the ways participants may engage in future interactions. Following this description, for the purposes of this study, two unique features characterized an episode: (1) it must be a verbal interaction, and (2) the interaction had to be about mathematics. I defined an interaction as a speech act where the intent of the speech action was taken up by other members and included dialogue from both the teacher and student. If Jamie was the only participant to speak, it was not categorized as an episode of interaction; both student and teacher must have a speech act. I used this criteria to define an episode of interaction because I wanted to understand *shared* meanings of teaching and learning and thus, a speech act from both teacher and student were required to mark an episode of interaction.

Second, the interaction had to be about mathematics. During 90 minute long class periods, there were occasional conversations between teacher and students that were not about mathematics; some were about discipline, some about assignments from other classes, and some were about social lives outside of school. I made the decision to only consider episodes of interaction about mathematics because I wanted to understand the

meaning of teaching and learning in the specific context of mathematics—issues of classroom management and social calendars did not align with this goal and were omitted.

Another issue around defining episodes included determining the beginning and end of an episode. When launching tasks and leading discussions, episodes began when a mathematical idea was introduced and taken up by both teacher and student, and ended when the subject of that conversation shifted to another idea. For example, the teacher and students might begin by talking about a table of values, and once they shifted to discuss how to develop an equation for this table, the episode ended and a new episode began. When monitoring, episodes were denoted by Jamie's interaction with each group such that as she walked around the room, episodes were individually marked as she engaged with each group. However, teacher and student voice needed to be present; thus, if Jamie approached a group and told them to check their answer and no one in the group addressed Jamie, then there was no evidence of an episode of interaction.

Positioning Triads. Once episodes of interaction were identified, I openly coded each episode of interaction using thematic analysis (Miles, Huberman, & Saldaña, 2014) to identify both the essence of the interaction and possible storylines of the meaning of mathematics communicated through episodes. Descriptive codes were used to describe the focus or topic of the data (Creswell, 2013; Miles, Huberman, & Saldaña, 2014), and themes were identified and coded, not for generalization purposes, but rather to understand the true nature and complexity of this case (Creswell, 2013). Open-coded themes included organization, agency, precision, authority, and competency. These

overarching themes helped me to understand the broad nature of storylines within the classroom space.

Next, positioning triads were created for each episode to support storylines initially identified. Within positioning triads, I coded using the three constructs from Positioning Theory: storylines, positions, and communication acts. Positions assigned and/or assumed by teacher and student included *authority*, *questioner*, *sense-maker*, *listener*, *follower*, and *facilitator*. The positions were fluid in the sense that both teacher and student could assume each position at some point within the lesson under examination. I identified positions and used them to support, challenge, or refute previously identified storylines. While I acknowledge there are a variety of storylines present in any episode, I worked to capture the overall essence of the episode, and therefore, I tried to identify one teaching and one learning storyline and one position for teacher and one position for student for a total of four codes per episode. While the goal was to identify only one code, this proved to be much more complex, and thus, many episodes were coded more than four times. Some episodes were coded with multiple storylines and others were coded with multiple positionings for teacher and student. For example, there were many positionings where a teacher was coded as being both a *questioner* and *authority* within a storyline of teaching mathematics.

To illustrate, the following episode occurred spring 2017 as students engaged with a task of maximizing area. This episode provided evidence of multiple positionings for Jamie and her students as well as multiple teaching and learning storylines.

Jamie: Yes. So the square pen would be it. All right. So then we have Casey.

Student: Yay.

Jamie: So Casey. You want to talk a little bit about what you did?

Student: Well, first we tried to find the function and that proved difficult so seeing is how by the end we got zeros of $Y = 0$ and then $X = 36$ and directly between those would be the line of symmetry which would be a square $X = 18$, which also is connected to the vertex. So we tried 18 as in that table over there [gestures to another groups' table of values on the board] and found we could [inaudible] with that being the vertex.

Jamie: Okay. So how did you come up with this because I know that we had a conversation about coming up with your functions so kind of where — first of all, how did you know it was gonna be quadratic?

Student: It's what we've been studying recently.

Jamie: That makes sense. Okay. So we'll go back to the problem and talk some more about that in a minute. So he figured we've been doing quadratic functions so this is probably quadratic so let me try it. So what made you decide on this?

Student: Well, seeing as how all the sides we had are 72 then the length and the width had to add up to 36.

Jamie: All right. So does everybody see that? Where he got his 36?

Student: Yes

Jamie: And then I did kind of help him recognize that these needed to be negative because when he factored it, he got 0 and then he got -36 and we said wait, you can't have a length of -36. So he multiplied the entire thing by -1 to get his two factors, okay?

Instead of four codes, this particular episode was coded seven times; two teaching mathematics storylines, two learning storylines, one teacher positioning, and two student positions were coded. First, *teaching meant eliciting kids' thinking* and second, *teaching meant making connections* between students' thinking. While the latter is not as evident

in the dialogue, during the video, Jamie worked to highlight other students' work that was presented and displayed earlier in the discussion during her final talk turn, such that teaching mathematics came to mean making connections. Similarly, there were two learning storylines coded. First, *learning meant communicating thoughts and ideas* and second, *learning mathematics meant sense-making* and problem solving. Jamie's reflexive positioning was coded as *facilitator* and students were interactively positioned as *authority* and *sense-makers*. I provide this example to call attention to the complexities of coding episodes of interactions and challenge the idea that all episodes were indicative of one teaching storyline, one learning storyline, one teacher positioning, and one student positioning, because they were not. Instead, often episodes positioned teachers and students in a variety of ways and followed multiple storylines around the meaning of teaching and learning mathematics.

Video-Stimulated Recall Interviews. I conducted video-stimulated recall interviews (Appendix A) to understand Jamie's perception of storylines and positions in her classroom as well as the relationship between reflexive and interactive positioning. During these interviews, Jamie and I viewed carefully selected episodes of interaction across all four lessons. I selected and pulled episodes of interaction using a few criterion. First, episodes selected were coded with multiple teaching and learning storylines and multiple positions for teacher and student. I chose these episodes for member-checking purposes to push or confirm storylines and positions identified. Second, episodes were chosen that represented both a success and struggle for Jamie in her practice. Both criteria were necessary such that Jamie could see herself in a positive light, but also

note the progress she had made during professional development and also so I could become clearer in the meanings of teaching and learning in this classroom space. During interviews, the use of language altered slightly. Using a grounded approach, Jamie and I used the term “roles” to describe teacher and student positioning and “narratives” to describe applicable storylines. While as a researcher I recognize the distinction between roles and positions, I also acknowledge the need for a safe and open space for Jamie. Thus, with the purpose of establishing open and honest dialogue with Jamie, we utilized phrasing meaningful for her.

Four video-stimulated recall interviews were conducted during summer 2018, and Jamie retroactively viewed her lessons from spring 2015, spring 2016, spring 2017, and spring 2018. Interview one focused on storylines (narratives) and communication acts across all enactments. Interview two concentrated on positions (roles) and communication acts across all enactments, interview three on the relatedness of teacher and student positioning, and the final interview clarified definitions, statements, or comments Jamie used in previous interviews. Each interview was recorded, field notes were collected, and to aide in analysis, each interview was transcribed and memos were made in margins.

Transcripts of video-stimulated recall interviews were analyzed to member-check Jamie’s view and interpretation of findings (Creswell, 2013). Specifically, initial interviews (VSR interviews 1 and 2) presented Jamie with episodes of interaction from each of her four lessons. For each time point, Jamie reflected on communication acts, positions (roles), and teaching and learning storylines (narratives) referenced. I presented

findings from my analysis and I made note of confirming or non-confirming evidence and encouraged alternative language or stance on storylines and positions I had identified (Stake, 1995). Jamie was open, honest, and tasked with disagreeing or challenging positions or storylines she felt were inaccurate or insufficient. These interviews were used to consider rival hypotheses and alternative interpretations of storylines and positions coded during each classroom enactment. When Jamie agreed or disagreed with the findings or interpretation, I honored her response and reported her feedback.

The third video-stimulated recall interview was used to understand Jamie's perception of positioning and the related nature of teacher and student positioning. This VSR interview took place in summer 2018 as well, and occurred after member-checking had been completed and we had arrived at a shared understanding of the positions (roles) and storylines (narratives) present during previous enactments. During the fourth and final interview, I clarified some of the language Jamie used, asked her to consider how some of her roles were similar and different from one another, and asked clarifying questions to ensure I was accurately reporting her perceptions of relatedness in positions.

Strategies for Validating Findings

Yin (2013) suggests the use of rival explanations as an analytic strategy for case study researchers. In acknowledging researcher positionality and bias, I considered rival storylines and positions from the participant's vantage point within episodes of interaction. Findings, therefore, did not rely solely on the interpretation of the researcher, but on the shared understanding between researcher and case study participant.

Maxwell (2013) defined validity as a means of drawing conclusions or explanations and ensuring the credibility of each. Thus, to enhance the validity of my research I utilized many of Creswell & Miller's (2000) eight considerations for validating qualitative research. First, long-term involvement over the course of three years with the case served as one validation measure. I attended each professional development session during three years (270 hours), recorded all classroom lessons to be analyzed (12 hours total and 6 hours reported in this study), attended three professional conferences with Jamie, and had numerous informal conversations. Second, using evidenced mathematics classroom storylines and positions found in relevant literature, I triangulated findings and methods to serve as a baseline. Also, I employed member-checking measures with Jamie to consider alternative interpretations of storylines and positions as well as consider alternative language and meaning.

Finally, Creswell & Miller (2000) encourage researchers to consider negative-case analysis. I chose Yin's (2013) notion of "rival explanations" instead to increase the credibility of this case study (Appendix B). I used and considered a variety of possible rival explanations for my results, which were also discussed at length with Jamie during video-stimulated recall interviews. Three of the more pressing explanations discussed with Jamie included the notion that (1) storylines were perhaps a product of a wider school culture and not unique to her classroom, (2) the possibility that these particular storylines and positions were only observed during these lessons and were not part of her typical classroom climate, and similarly (3) storylines and positions could be explained by task implementation and therefore were not standard in her classroom lessons.

Positionality of Researcher

As researchers, we are merely instruments that have been molded, shaped, and impacted by our experiences, developing a unique lens through which we view the world around us. As a former secondary mathematics teacher turned researcher and doctoral student, I understand firsthand the complexities of classrooms, school systems, and educational culture. As a teacher, I experienced the stress of making sense of new mathematics standards, making in-the-moment decisions about student thinking, questions to pose, lesson progression, and the sensitive nature of developing rapport and relationships with my students. My role in this study was two-fold. In addition to being the researcher in this classroom, I also served as a facilitator of the professional development around high-quality mathematics core instructional practices in which Jamie has participated for the past three years. Thus, I was simultaneously researcher and facilitator.

In addition to those two roles, I had other experiences that granted unique insight into this particular context. First, my former work as a mathematics educator was in the same school in which I studied, so Jamie is a former colleague of mine and we worked closely together while teaching Math 1 and eighth grade mathematics. Though this could be perceived as an issue affecting my ability to be unbiased and impartial, I argue that my experience as a teacher in the school and colleague of Jamie's granted me "insider status" so that Jamie felt comfortable opening her classroom door to me, which allowed unique perspective and invaluable insight into her classroom. Given our past, Jamie was comfortable to be herself during recorded observations rather than feeling the need to put

on a “show” of what she thought I wanted to see. Similarly, honest and raw rapport was established such that during video-stimulated recall interviews, Jamie had a safe place and a safe person with whom she could authentically share her thoughts and feelings. I acknowledge that while our history and established relationship foregrounded particular ideas and opened the door to conversation, it also created blind spots in this research. Video-stimulated recall interviews were an attempt to mitigate such blind spots as I asked Jamie during each interview what stood out and was most interesting to her about her practice and interaction with students.

I am interested in storylines, positioning, and the related nature of positioning from Jamie’s perspective, so I was purposeful in honoring Jamie’s voice and opinion. My goal was to present Jamie and her students in a positive, strengths-based manner, while also highlighting the complexities of mathematics teaching and learning. It was my hope that throughout the findings and discussion, Jamie’s commitment to students, learning, and professionalism were foregrounded and evident and yet, as educators, we can pinpoint the difficulties in this work and locate ourselves and our struggles in aspects of Jamie’s experience and dialogue as we all strive to develop our own mathematical practice.

CHAPTER IV

FINDINGS

The storylines that framed episodes of mathematics teaching and learning in Jamie's classroom shifted throughout her engagement in professional development in three ways; some storylines were consistent, others dissolved, and still others emerged. First, there were communication acts and related positions that persisted and were relatively consistent throughout her instruction over three years and resulted in little, if any change in storylines and the meanings of teaching and learning mathematics. Other storylines were present initially, but over time as communication acts and positions dissolved some storylines were no longer present. Finally, there were a variety of storylines that mirrored new norms of interactions and positions and thus, new storylines emerged as Jamie engaged in sustained professional development and reflection on her practice. Initially, learning mathematics meant students listen and follow directions as teaching mathematics meant a teacher telling. Over time however, teaching mathematics came to include listening and learning mathematics suggested students tell and explain. In what follows, I organize and present findings by three types of change. I organize based on shifts in storylines – those storylines with limited change, those that were no longer present, and finally those that emerged over time. Shifts in storylines necessarily imply altered speech acts, norms of interactions, and positions of participants.

However, I chose to organize by shifts in storylines first and within each shift, describe more nuanced shifts in communication acts and positioning. Six positions were identified and included: *questioner*, *follower*, *authority*, *facilitator*, *listener*, and *sense-maker*. While these positions were not quantitatively assumed equally, both student and teacher participants had access to each position.

Given the reciprocal and mutually determining nature of teaching and learning in a classroom, all episodes from my analysis followed and were coded for at least two storylines, one about the meaning of teaching mathematics and another about the meaning of learning mathematics in Jamie's classroom. Thus to share my findings, I describe most often paired teaching and learning storylines to preserve the complexity of positioning as well as honor the relational nature of positioning and the variety of vantage points with which to enter the work. However, the pairings are not unique and one-to-one pairings. That is, different teaching and learning storylines were matched throughout episodes of interaction over time and across lessons.

	2015	2016	2017	2018	Teacher Position	Student Position
Persistent Storylines						
Teaching means telling.	7	3	5	4	Authority	Follower
Learning means listening.	6	3	9	4	Authority	Follower
Teaching means eliciting student thinking.	4	4	4	2	Questioner	Follower
Learning means listening.	6	3	9	4	Authority	Follower
Dissipated Storylines						
Teaching means equitable access.	1	2	1	0	Questioner	Follower Sense-maker
Learning means having a starting place.	2	2	1	1	Questioner	Follower Sense-maker
Teaching means organization.	4	1	0	0	Authority Questioner	Follower
Learning means organization.	3	1	0	0	Authority Questioner	Follower
Emergent Storylines						
Teaching means pressing students to justify.	0	4	8	4	Questioner	Authority Sense-maker
Teaching means facilitating discussions.	0	2	2	2	Facilitator Questioner	Authority Sense-maker
Teaching means making connections.	0	1	6	2	Facilitator Questioner	Authority Sense-maker
Teaching means listening.	0	0	4	2	Facilitator Listener	Authority Sense-maker
Learning means communicating.	0	4	7	2	Questioner	Sense-maker Authority
Learning means asking questions.	0	2	2	0	Facilitator Listener Questioner	Sense-maker Questioner
Learning means sense-making.	0	0	7	3	Facilitator Questioner	Authority Sense-maker

Figure 3. Teaching and Learning Storyline Counts and Positions Over Time

Persistent Storylines

I begin with teaching and learning storylines that remained relatively consistent throughout Jamie's lessons. These storylines as well as the nested positions within each

were constant and did not change throughout Jamie's participation in professional development. *Teaching means telling* most often occurred with a storyline of *learning means listening and following directions*. These storylines persisted were present in each lesson throughout the study from Spring 2015 through Spring 2018. Additionally, *teaching means eliciting kids' thinking* showed limited change and was paired with the aforementioned learning storyline around *listening and following directions*. Below, I use the positioning triad to describe storylines, related positions, and communication acts that structured interactions between Jamie and her students around mathematics and offer examples from across lessons to show how Jamie and her students' patterns of communication worked together to reproduce a meaning of mathematics teaching and learning consistent with a direct model of instruction. I then present representative episodes from Spring 2015 and Spring 2018 to illustrate how communication acts and positions consistently reproduced storylines of teaching and learning mathematics that meant telling and listening.

Teaching means Telling and Learning means Listening

Two of the most often referenced storylines during Jamie's early episodes of interaction were the ideas of *teaching means telling* (19/62) and the complementary storyline of *learning means listening and following directions* (22/62). Episodes following these storylines were marked by a variety of communication acts but limited positions for teacher and students. In a Spring 2015 episode for example, when students asked how to proceed, Jamie responded with, "*I'm going to help you*" and other times suggested, "*so, I'd draw it again. That's what I did when I was doing this problem*", and

“All you do is multiply that number and double the zeros”. When mathematical misunderstandings arose, incorrect or incomplete processes were utilized, such as multiply and add zeros, and when Jamie seemed unsure about students’ mathematical thinking and reasoning, she regularly began by telling and explaining how to move forward. By telling and explaining the mathematics, Jamie reflexively positioned herself as a *mathematical authority* in the room while interactively positioning students as less powerful *followers*; students, rather than contesting positions from Jamie, compliantly accepted positions of *followers*. Thus, what it meant to learn was marked predominantly by teacher voice as students were led through a series of steps toward a solution such that students’ noticings and wonderings were often silenced. Positioned as *followers*, students often said “okay”, nodded their heads in agreement, wrote down notes, and looked on another students’ paper if Jamie was referring to written work. Students were expected to listen to mathematical explanations, answer directive questions when prompted, and follow Jamie’s mathematical lead suggesting *learning meant listening and following directions*.

Not only was *teaching means telling* and *learning means listening* most often supported in interactions occurring in 19/62 and 22/62 total episodes respectively, they were also consistently present in each lesson over four years. *Teaching means telling* occurred in 7/15, 3/13, 5/25, and 4/9 episodes, which highlights the predominance and persistence of this storyline. Similarly, *learning means listening* occurred in 6/15, 3/13, 9/25, and 4/9 episodes of interaction. While both storylines were consistently assumed,

they were not always paired. *Learning means listening* was also paired with a teaching storyline of *teaching means eliciting kids' thinking*.

Teaching means Eliciting Students' Thinking and Learning means Listening

Another storyline throughout Jamie's lessons was *teaching means eliciting kids' thinking* (14/62), which also frequently occurred in episodes with *learning means listening and following directions* (22/62). *Teaching means eliciting kids' thinking* was another consistent storyline evidenced as Jamie posed questions or made declarative statements aimed to uncover student thinking and get students talking. For example, Jamie's speech acts in Spring 2015 included, "*where did you get this*," "*what did you come up with*," "*Tell me what you did here*", etc. As Jamie reflexively positioned herself as a *questioner* to elicit thinking, questions and statements posed established underprivileged positions for students of *follower* by limiting their opportunities to mathematically reason or make sense. *Teaching means eliciting kids' thinking* often positioned Jamie as *questioner* of students and students as *followers* as they listened to directions and responded to surface level prompts of their thinking.

Episode of Interaction

Above, I defined storylines and provided quantitative evidence of change over four years; in what follows I provide further qualitative evidence to illustrate how those storylines and positions remained consistent throughout the study. I provide two episodes, one from Spring 2015 and another from Spring 2018, to highlight *teaching means telling* and *eliciting kids' thinking* and *learning means listening*. I begin each episode with a short description of the context of the lesson and classroom environment

before providing the episode transcript. I end with a short analysis of the episode of interaction and storylines and positions coded within each.

Spring 2015. Prior to participation in professional development, Jamie's colleagues and administrative team would describe classroom management as a strong point of her teaching. Jamie's classroom was so tightly structured and managed, it seemed regimented. While there, I noticed students consistently raising their hands before sharpening pencils, being dismissed by tables to get calculators, and Jamie circulating the room and working with groups in a particular order. During spring 2015, students were presented with the following task to solve:

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower, that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Most students in Jamie's Math 1 class used a mixture of Pythagorean theorem and a guess and check approach to find the minimum amount of wire. In Spring 2015, teaching math meant telling and learning meant following directions. For example, consider the following episode where Jamie engaged with a group of students who started by placing the island in the center of the lagoon at (300, 300) to find the total length of zip line wire.

Jamie: Alright, so what have you guys figured out so far?
[Student explains the work they have been doing to place the island in the middle of the lagoon at 300, 300 and then says they are unsure what to do next.]

Jamie: So, move your point. Which way do you want to move it?

Student: Left.

Jamie: Okay. So –

Student: But how do we figure out –

Jamie: I'm going to help you.

Student: Okay.

Jamie: So, I'd draw it again. That's what I did when I was doing this problem.

Student: Okay.

Jamie: And you just get to the side, so if you're going to move it to the left, what do you want that left distance to be? Right now, it's 300. If you adjust it, what are you going to make it?

Student: 200.

Jamie: Okay, so make that 200. Which means the other one has to be?

Student: 400.

Jamie: Okay. So, now figure out how long that wire is.

While monitoring small group work, Jamie approached this group of students and said, “*What have you figured out so far?*” “*move your point*”, “*I'd draw it (the diagram) again.*” These communication acts positioned Jamie as an *authority* and students as *followers* as they followed her directions. However, as the episode progressed, her speech acts consistently focused on *eliciting* student thinking in such a funneling manner

as to lead students toward particular solution methods. These communication acts and positions did not push students in their thinking, alternative solution strategies were not considered, and justification of thinking and reasoning was not normative practice. Once the student had explained their thinking to that point and where they were in the process, Jamie may ask another question, but often worked to explain next steps and what should be done to move forward (“*what do you want to make it?*” “*which means the other has to be?*”, and “*now figure out how long that wire is?*”). While initially beneficial in creating student discourse, such communication acts positioned students in ways that limited their opportunities to reason and make sense of the mathematics. Thus, *learning meant listening and following directions while teaching mathematics meant telling and eliciting kids’ thinking.*

Other non-verbal communication acts during this lesson also indicated Jamie’s authoritative position and students as followers. While Jamie circulated the room, disconnect and distance between Jamie and her students was evident physically and verbally. Physically, Jamie clutched a clipboard to her chest she used to make note of student thinking and approaches. The clipboard made her unapproachable, creating a barrier to interaction from students, as her gaze was fixated on a clipboard instead of looking around making eye-contact with students. Even Jamie seemed to intrinsically pick up on the distance created by a clipboard and would often lay it down when engaging with small groups in conversation. Jamie verbally communicated separation between herself and students as well. In the episode above, her language and pronoun usage were exclusive in nature. She asked students what “*you guys*” have figured out,

which way “*you*” want to move it, and makes the statement that *she* will help *them*. Her use of pronouns further supports and articulates the divide between teacher and student; students were expected to investigate the mathematics and her job was to help students as necessary because she held the necessary mathematical knowledge. Across the episodes in Spring 2015, there were rarely instances of inclusive “we” pronouns to indicate teacher and students as a collective unit or team when approaching mathematics.

Summary. Storylines most often referenced in early (Spring 2015 and Spring 2016) lessons suggested Jamie’s initial conception of mathematics was *teaching meant telling* (10/28), and *teaching meant eliciting student thinking* (8/28) where Jamie most often positioned herself as *authority* and *questioner*. Following Jamie’s lead and coordination of the meaning of teaching mathematics, the meanings made of mathematics learning centered on *listening and following directions* (9/28) that positioned students as *followers* with limited opportunities to make sense. When assuming a *questioner* position, Jamie would uncover student thinking and guide them through a series of steps moving forward. Jamie reflexively positioned herself as an *authority* through her communication acts of telling and explaining procedures and steps moving forward. Jamie’s positioning of *authority* was not only evidenced in her teaching storyline and verbal communication acts, but also in her non-verbal social cues.

Spring 2018. Following three years of professional learning, some episodes of interaction carried the same meanings and storylines of teaching and learning mathematics, namely *teaching means telling* (4/9) and *learning means listening and following directions* (4/9) were storylines that were still evidenced in interactions.

During spring 2018, students were given a series of smaller tasks to conceptually make sense of inequalities and the implications of integers on inequalities. Below, Jamie and her class discussed how to graph the following inequality: $4s + 6 \geq 6 + 4s$.

- Jamie: So what could s be to make this true?
- Student: Anything.
- Jamie: Anything. So your solution would be what?
- Student: Anything.
- Jamie: Or all –
- Student: [Inaudible]
- Jamie: Oh, yellow's not good. Or all what?
- Student: Real numbers
- Jamie: Yeah, so all real numbers. How would you graph that one a number line? It's just the entire thing.
- Student: One thing with a little circle on the end.
- Jamie: It's just the entire number line. It's all real numbers, there's no restrictions. It would just be the whole number line. How would you write that in interval notation?
- Student: [Inaudible] [00:23:14] to infinity.
- Jamie: Yup, so parenthesis, negative infinity, comma, infinity, closed parenthesis. And then set notation would just be s – this is the weird one – s such that s this means is the set of all real numbers.

This episode suggested storylines of *teaching meaning eliciting kids' thinking* as well as *teaching means telling*. Jamie's speech acts, "or all what?" "how would you graph that

one...” and “*how would you write that...*”, encouraged students to share their mathematical thinking but did not press students to consider why all real numbers was an appropriate solution, what all real numbers even means, or given a context with which to create, grapple, or make sense of this particular inequality. Such storylines and communication acts once again positioned Jamie as an *authority* and students as *followers*. This particular episode highlights that when *teaching meant telling* and *eliciting kids’ thinking*, student’s response was obligatory and lacked mathematical depth and reasoning. Hence, the meaning of learning mathematics still circulated a storyline of *listening and following directions* from a more knowledgeable other.

Summary

Some of the meanings made of teaching and learning mathematics in Jamie’s classroom remained the same throughout Jamie’s lessons over four years. Those storylines with limited changes about teaching mathematics included *teaching means telling* and *teaching means eliciting kids’ thinking*. With these teaching storylines, Jamie reflexively positioned herself as a mathematical authority with the understanding and know-how to proceed and make sense of the task. Thus, learning mathematics came to mean *listening and following directions*, and as Jamie assumed a position of *authority*, Jamie often interactively positioned students as mathematical *followers*. By describing these storylines as having limited change, I mean that these storylines of teaching and learning were consistently present in each lesson from spring 2015 to spring 2018. These storylines make sense and align with cultural, societal, and educational norms around

teaching and learning and the ways we often believe learning is accomplished in classrooms.

As Jamie engaged in sustained professional development however, what it meant to teach and learn mathematics in her classroom changed. As we would expect, *teaching means telling* and *eliciting kids' thinking* and *learning means listening* were storylines that were so engrained in Jamie and her students that they persisted across time, but the nuanced timing of when to tell and when to elicit shifted. While *teaching means telling* and *eliciting kids' thinking* and *learning means listening* were storylines that were present in each year of the study, other storylines that did not withstand the test of time and were not as inherent in Jamie and her students meanings of teaching and learning, as well as those meanings of teaching and learning that emerged. In what follows, I first discuss storylines of teaching and learning that went away and then move to describe storylines that emerged over time.

Dissipated Storylines

While some storylines were present in Jamie's classroom throughout the study, other teaching and learning storylines were no longer evident in episodes after participation in the PD. Those storylines included *teaching means organizing*, *teaching means equitable access* for students and the complementary learning storylines of *learning means organizing work* and *learning means having a starting point*, respectively and dissipated as Jamie progressed with her involvement in professional development and were only present in Spring 2015, 2016, and some in Spring 2017. By Spring 2018, these storylines were no longer observable and seemed to have become more engrained

and absorbed within classroom norms, and thus, were no longer observable in Jamie's practice and interactions with students. In what follows, I discuss all constructs of positioning triad to highlight the meaning of teaching and learning mathematics. In what follows, I first discuss teaching and learning means organization and then move to teaching and learning centered on access and getting started. I conclude with an episode of interaction to assist the reader in grounding teaching and learning storylines of organization and access within interaction episodes.

Teaching and Learning mean Organization

Some episodes from spring 2015 followed a storylines where *teaching means organization*, such that Jamie helped students organize their thinking and work in meaningful ways. Similarly, initial episodes suggested *learning meant organization* as well. With a narrow and unwavering focus on organization, Jamie encouraged students to keep record of their thinking in universally understandable ways such that when she approached a group, she could quickly make sense of the work on the paper. Communication acts such as, “*You’ve quit writing down your data, so I don’t know where you are,*” “*I would put those two [columns] and then the final outcome [column],*” and “*the next thing you need to do is figure out how you are going to organize your data*” pushed students to model their data in an ascending/descending table of values, which suggested both teaching and learning mathematics was about organization. These statements made to both whole and small groups positioned Jamie as an *authority*, while students compliantly assumed *follower* positions heeding her direction to organize their thinking in ways that made sense to Jamie. While creating a table seemed beneficial for

some groups of students, the vast majority of groups seemed content and able to make sense of their work while displaying data in a variety of other ways, suggesting organization was for Jamie's benefit instead of students. Teaching and learning storylines centered on organization positioned Jamie as an authority and students as compliant.

Organizational teaching (4/15) and learning (3/15) storylines were paired almost exclusively with one another and both storylines were present prior to the professional development provided by our research team in spring 2015. After just one year of sustained professional development around core instructional practices, organizational teaching and learning storylines were only coded in one episode. By spring 2017 and spring 2018, organization was no longer a storyline called forth in Jamie's classroom. Teaching and learning mathematics had shifted toward more holistic, sense-making practices discussed later.

Teaching means Equitable Access and Learning means Having a Starting Place

Early episodes in Jamie's classroom included a meaning of *teaching mathematics means equitable access*—Jamie ensured all students had a starting point and could immediately dig in and begin grappling with the task at hand. Having a starting place did not mean that students knew the answer or even the steps or processes necessary to get an answer; rather, *learning mathematics meant having a place to start* and understanding the contextual elements of the task at hand. A starting point could be as simple as drawing a diagram, but students' entry into mathematics was open-ended, where a certain strategy was not prioritized, and was marked by multiple entry points. To accomplish this, Jamie was purposeful in granting time for students to think, consider, and ask questions before

they started working in groups. Her communication acts followed an almost prescriptive format during this storyline—students were asked to put their pencils down individually consider the problem (“*Alright guys, no pencils, just think for a minute.*”), collectively (in partners or small groups) discuss ideas (“*Now, turn to a friend and tell them how you are thinking about this problem.*”), and then, as Jamie circulated the room, students worked collaboratively to get started. *Teaching means equitable access* often positioned Jamie as questioner and students as mathematical agents tasked with making sense.

Summary

Storylines of the meaning of teaching and learning mathematics in Jamie’s classroom shifted throughout her engagement in sustained professional development. Teaching storylines of *organization* (5/28) and *equitable access* (3/28) and their corresponding learning storylines of *organization* (4/28) and *having a starting point* (4/28), were initially important to teaching and learning in Jamie’s room during Spring 2015 and Spring 2016 lessons. However, as Jamie’s practice changed, so too did those storylines of teaching and learning, such that organization and having a start place were no longer referenced and called forth during interactions. This is not to say that these storylines were no longer valuable in this classroom space, rather, these seemed to become embedded more within classroom norms and less explicit in practice.

Emergent Storylines

While some storylines persisted throughout lessons and some storylines dissipated, there were a variety of storylines that emerged as Jamie’s practice and work in the professional development continued. There were four teaching and three learning

storylines that surfaced as Jamie engaged in professional learning. These storylines were not present in episodes from the Spring 2015 lesson prior to professional development but emerged in episodes during the Spring 2016, 2017, and/or 2018 lessons. These new teaching storylines included *teaching means pressing kids to justify*, *teaching means facilitating discussions*, *teaching means making connections*, and *teaching means listening*. Similarly, there were new learning storylines to emerge including, *learning means communicating*, *learning means asking questions*, and *learning means sense-making*. With emergent storylines, new positions were assumed by Jamie and afforded to students during interactions. In what follows, I organize by teaching storyline, describe all learning storylines coded with it and then move to discuss teacher and student positions afforded and assumed.

Earlier storylines of teaching and learning that persisted in Jamie's classroom or dissolved were organized in teaching and learning pairs. While teaching and learning pairings were not always exclusively paired, the pairings discussed above were more consistent and frequent throughout interaction episodes. Moving forward, pairing teaching and learning storylines one-to-one is not sufficient for a variety of reasons, but most importantly, there was significant "overlap" between storylines. For example, *teaching means facilitating discussions* does not easily pair with any one learning storyline. Rather, depending on context and positioning of teacher and student in interactions *teaching means facilitating discussions* could elicit a storyline of *learning means sense-making* or even *learning means asking questions*. Thus, for this section, I

begin with teaching storylines; discuss the multiple learning storylines elicited within each and the represented positions of teacher and student within enacted storylines.

Teaching means Pressing Students to Justify

As Jamie worked on her practice, one of the more striking differences was in her questioning strategies and timing of questions. Earlier, I discussed a storyline that was present throughout all of Jamie's lessons and that was that *teaching means eliciting kids thinking*. While this particular storyline was present consistently throughout Jamie's lessons, Jamie's practice evolved to include pressing students' thinking forward through justification. More often than not, *teaching means pressing kids to justify* storyline was complemented with a *learning means communicating* storyline. Such teaching and learning storylines one again positioned Jamie as a *questioner*, which was a position previously assumed, but while her positioning of *questioner* was consistent, the nature of the storyline and questions was markedly different. As a *questioner*, Jaime pushed students to consider new ideas by interactively positioning them as *authority* and *sense-makers* and suggested a storyline of *learning means communicating* your thoughts and ideas. Students were now offered previously unattainable positions of authority and sense-makers as they engaged in mathematical discourse *with* Jamie. Having authority as mathematical sense-makers promoted student's privilege and status within the classroom space and supported the storyline of *learning means communicating* and *teaching means pressing students to justify*.

In what follows, I provide a brief, small group episode of interaction to capture the nature of *teaching means pressing kids to justify* and *learning means communicating*

thoughts, as well as provide the reader a sense of authoritative positioning for students since this is a newly afforded position. This episode is from spring 2018 as students work to make sense of inequalities. I use this episode to underscore Jamie's questioning and students verbalizing their mathematical reasoning and justification.

- Jamie: So, if it's a lower negative number? So, can you give me an example of a lower negative number?
- Student: Like -8.
- Jamie: So then what would be a higher negative number?
- Student: Like -1 or -2. Oh no, because then it still wouldn't work. I guess it's just a negative number then.
- Jamie: What do y'all think about what Devion is saying?
- Student: I agree with him.
- Jamie: Why do you agree with him?
- Student: **[Inaudible] [00:07:45]**
- Jamie: But he was saying something about that there's maybe there were some negative numbers that could and maybe some negative numbers that couldn't work. I think he's talked himself out of it, though. But what do you make of all that?
- Student: I don't think it works anymore.
- Jamie: So do you think it's all negative numbers?
- Student: Yes.
- Jamie: What do y'all think?

Jamie's questioning not only supported this student in justifying his thinking, but also invited others in the group to consider his ideas and build (i.e. "*What do y'all think about what Devion is saying?*" "*Why do you agree with him?*", and "*What do you make of all of that?*"). Further, Jamie pushed students to take a stance and articulate the *whys* behind their reasoning. Such communication acts by Jamie supported students in articulated thoughts and ideas positioning them as *sense-makers*. Thus, *learning was communicating* across multiple audiences including student-to-teacher and student-to-student. Next, I build on this communication to include communicating thoughts and ideas with peers.

Teaching means Facilitating Discussions

Another storyline that emerged over time focused on generating discourse amongst students during small and whole group discussions. Generating discourse pushed a new narrative of teaching and learning and required Jamie to reflexively and interactively position in new ways. When teaching mathematics grew to mean facilitating discussions, Jamie assumed a new position as *facilitator* and students were positioned as *sense-makers* with responsibilities to hear, grapple, make sense, and cultivate connections among their peers thinking. Thus, the meaning of learning came to include not only sense-making, but also *learning mean asking questions* to solidify and build partial understandings. Jamie's *facilitator* positioning was evident in limited communication acts because often, facilitating meant *not* speaking. As a communication act, not speaking pushed students into positions of sense-making. However, when Jamie spoke speech acts included, "*So what made you decide on this?*", "*I mean, you can see it*

but do y'all see the pattern here?", "What do you notice?", "What do you see?", "Which means what?", "But what's different about what Caleb did?", "Kyle, what did you see?". As Jamie assumed a facilitator position, students were encouraged and supported in assuming positions of authority and sense-maker and they regularly accepted.

Teaching means Making Connections

After discussions were initiated whole group and small group among students in the classroom as discussed above, episodes would often shift to another meaning of teaching mathematics in which Jamie made connections across student thinking. Initially, this work fell primarily to Jamie as the meaning of learning that was taken up still rested with sense-making and asking questions. Notably, one staggering difference in learning came to include asking questions not just to solidify personal understandings, but also rather to make connections between work. Students seemed to mirror Jamie's modeling that if teaching were about making connections, learning mathematics too, must mean asking questions to support those connections. In the episode below, Jamie and her students engaged in a whole group discussion in spring 2017 centered on making connections. Central to this episode are the connections between student ideas and thinking, as well as connections to previous mathematical tasks and content. Both teacher and student worked to link thinking and experiences to the work at hand.

Jamie: So how does that relate to what Casey did?

Student: It's a quadratic function.

Jamie: And it's pretty much the same function that Casey came up with, right? Okay. So then you can factor it, what Casey did, find your zeros, axis the symmetry and use that to find that maximum area.

Student: Isn't this like the one about the soccer field?

Jamie: This is exactly like the one about the soccer field. I'm glad you brought that up. Do y'all remember the soccer field problem?

Student(s): Yes.

Jamie: Okay. All right. So let's look back at the problem real quick and I want you guys to tell me what in the problem could have triggered you to say quadratic. Sophie, what are you thinking?

Students: I didn't quite get there but I figured it was quadratic because everyone — we are all trying to find —it reminded me of factoring it. If you're trying to find two numbers that like added to 72 and like multiply to make **[inaudible]**.

Jamie: Okay. Kyle, what'd you see?

Student: I think I got that **[inaudible]** Misha has purchased a 72 foot roll of fencing to buy a rectangular pen. If Misha wants Rascal to have the most room possible. And I started to think back to the soccer field when she was looking for the most possible area on the field technically.

Jamie: Most room possible...I intentionally did not use a certain word but most room possible. That means you're finding the what?

Student: Maximum.

The episode began with Jamie asking the class “How does that relate to what Casey did?” and moved to included questions of “Kyle what did you see?” and “Sophie, what are you thinking?”. What is more, Jamie worked with students to make a connection to previous learning tasks involving a “soccer field problem”. Such communication acts suggested a storyline of *teaching means making connections*, which often positioned Jamie as a

facilitator. The student's responses of explaining their thinking, why quadratics makes sense, and responding to Jamie with questions, "Is this like the soccer field problem?" suggests students assumed positions of sense-makers and listeners. As the episode progressed, Jamie's commitment to building student thinking from student thinking and using it as a springboard for further thinking and discussion was clearly evident. Similarly, storylines of learning mathematics held tight to sense-making, not only of what Jamie discussed as was the case in earlier lessons, but rather, sense-making of other students thinking and making connections among ideas. These storylines of teaching and learning being centered on making connections were not present in spring 2015, but emerged as Jamie worked on her practice.

Teaching means Listening

After Jamie worked to facilitate discussions and make connections among students thinking and responses, Jamie called forth storylines around *teaching means listening* where she assumed positions of *listener* or *facilitator*. Jamie's new position of listener was difficult to capture in transcriptions because of limited speech acts, but the absence of speech acts was indicative of Jamie's new positioning. Students' communications acts included detailed accounts of their thinking and reasoning throughout tasks, such that students were positioned as *authority* and *sense-makers*. Jamie's non-verbal cues were also noted in analytic memos such as nodding her head, giving a thumbs up to students, or perhaps pointing to a student who was talking in a group such to orient students to each others ideas.

Initial lessons insinuated *learning means listening*, so this shift from *learning means listening* to *teaching means listening* was perhaps one of the more significant shifts in her classroom practice. If *teaching mathematics means listening*, then *learning mathematics means sense-making* and *communicating mathematical thoughts and ideas* and such learning storylines pushed students into more agentic and powerful positions in the classroom. *Authority*, *sense-maker*, and *questioner* were newly afforded positions for students as they worked to articulate, defend, question, and build on mathematical ideas.

Summary

Storylines that emerged throughout Jamie's lessons included *teaching means pressing kids to justify*, *teaching means facilitating discussions*, *teaching means making connections*, and *teaching means listening*. Such storylines introduced and supported Jamie in assuming new roles of *listener*, *facilitator*, and *questioner*. Similarly, learning storylines emerged suggesting *learning means communicating*, *learning means asking questions*, and *learning means sense-making*. While these emergent storylines were not easily paired with teaching and learning, the supporting storylines pushed Jamie and her students into new positions where Jamie assumed less power and authority, opting instead to listen to her students, thus encouraging student voice and affording more agentic positions for kids.

Summary

Storylines and positionings followed three trajectories in Jamie's classroom. Some storylines in Jamie's classroom remained consistent and stable throughout participation in the PD including *teaching is telling*, *learning is listening*, and *teaching is*

asking question to uncover kids' thinking. Other storylines became less explicit as *teaching and learning is organization* and *teaching and learning is having a starting place* became more classroom norms rather than enacted storylines. Most interesting though were storylines that surfaced throughout Jamie's participation in professional development around core instructional practices. *Teaching is asking questions to push kids' thinking*, *teaching is facilitating discussion*, *teaching is making connections*, and *teaching is listening* were storylines that emerged over time highlighting the meaning of teaching mathematics. Additionally, *learning is communicating thoughts and ideas*, *learning is asking questions*, *learning is sense-making*, and *learning is making connections*, conveniently reflected and reciprocated what it meant to teach mathematics in this space.

The introduction of new teaching and learning storylines granted novel positions for both teacher and student during interactions. Students were afforded agentic sense-making and questioning positions initially reserved for Jamie, while Jamie renegotiated her reflexive positioning and assumed new positions of facilitator and listener.

Relational Positioning

The purpose of this study was first to understand how storylines and positions change as a teacher engaged in sustained professional development and then to identify Jamie's perception of relatedness between interactive and reflexive positioning. Given the ways that positioning theory has often been utilized in the field to understand discourse and equity, I wanted to use the theory with Jamie as a lens to understand her perception of positioning and if she envisioned a connection between her reflexive

positioning and positions interactively afforded to students in her classroom. By amplifying Jamie's voice and perception of positioning in her room, Jamie may work more explicitly during interactions.

One consideration to be made during interviews involved language. Thus, we made the decision and referred to positions as roles teachers and students take on during an interaction. While I view roles as rigid and positions as more fluid, and thus fundamentally different, it was important to use language Jamie was familiar and comfortable with. Thus, using a grounded approach, the term "role" naturally came from our conversations was consistently used to describe positions of teacher and student.

Jamie viewed recorded lessons and generated her own list of roles for students and teacher. Jamie began by focusing on herself and categorized her roles at different times to include organizer, teacher (provider of knowledge), explainer, reasoner, summarizer, sequencer, disciplinarian, evaluator, justifier, and questioner and then most often labeled students as taking on a listener role. Jamie listed roles observed during episodes of enactment for Spring 2015, 2016, 2017, and 2018. After she listed all roles assumed by teacher and student during enactment episodes, she described briefly what each role meant to her and the ways the roles were different from each other.

I wondered about role preference or roles that Jamie wanted to take on and if all of these roles were intentionally assumed. In the excerpt below from the summer 2018 video-stimulated recall interview, Jamie described teacher roles she most wanted to embody.

Researcher: What roles do you want to take on now as a teacher? What would you say are the three roles you want to assume in your classroom?

Jamie: So, I guess first is observer, listener, and then that would lead into the questioner, as far as where they are and is there any more probing I need to do or is it time to start pushing them to [the goal] that day. And then you have to facilitate that discussion at the end where we bring everything together and make those connections.

After watching lessons over four years, Jamie felt strongly that her role as a teacher was to be an observer, listener, questioner, and facilitator. While she saw these four roles as responsibilities that students could assume as well, she felt that to be an effective mathematics teacher she must truly embody such functions. As an observer and listener, she wanted to make students' voices heard. By taking a step back and working the periphery, she described giving student voice more prominence to develop student agency. For Jamie, learning mathematics was closely aligned with discourse and talking about math; thus, Jamie stated students must be the ones verbalizing their thoughts for learning to occur. As students discussed their ideas, Jamie wanted to assume roles of questioner and facilitator, and while the nuanced nature of these roles varied depending on context, overall, she viewed questioner and facilitator roles as those that exposed and made clear kids' thinking, pushing thinking, and making connections amongst mathematical ideas.

In one interview, I asked Jamie to consider if assuming roles of listener, observer, questioner, and facilitator had implications for students:

Researcher: Do you think that you [as a teacher] assuming roles of listener, questioner, observer, facilitator—do you think that necessarily determines what roles kids can take on?

Jamie: I think it takes...I think it sets the expectation that they're [students] **not** going to be listeners and observers, like if I take that role then it forces them into a different role, so yeah I think it does.

Researcher: Are you saying that you and students are not going to be in the same roles simultaneously? But do you think everyone can play every role?

Jamie: Yeah, I mean especially with the whole and small group element, I mean I would hope that they were listening and observing each other and there are going to be times when, you know, I say and do things and they're going to be listening to me or observing what I'm doing on the board or you know whatever the case may be.

Jamie saw roles as mutually exclusive, wherein if she assumed a particular role, then students could not take on that role at the same time. Thus, as Jamie assumed an observer or listener role, students were restricted and must assume roles other than observer or listener. To be clear, there were other students in the room that could assume a role of listener with Jamie while another student talked, but for the purposes of this interview, we were only referring to participants within an episode of interaction. This meant a student must initiate a speech act, which Jamie encouraged students to do so that she could assume observer and listener roles. Jamie went on in the interview to clarify listener and observer roles as functions students could assume while she was explaining. Therefore, Jamie saw roles as fluid between both teacher and student, meaning teacher and students could assume any role, but not something that could be simultaneously assumed by participants in an interaction.

Researcher: So, then what roles what do you think the roles of listener, questioner, observer, and facilitator—what roles do you think those promote for kids to take on? If those are the roles that you are going to assume?

Jamie: I'm listening and observing then they're having to explain, reason, and justify because that's what I'm listening to and listening for. And then I think going along with the questioning, if I'm in the role of questioner, that's going to again require reasoning [from students], you know, possibly explaining and justifying again to answer my questions. Then hopefully, even when I'm in the role of a facilitator, that's not going to be a time when I'm at the front of the room dictating, like I'm still in the role of facilitator, so I'm going to be asking them questions and expecting them to reason and make connections and be able to explain or justify, so I mean I think that explainer, reasoner, and justifier shows up like in a cycle through that whole teaching process.

Jamie was also explicit about roles she found most advantageous for students to assume when working and learning in mathematics. Jamie felt that for learning to occur students needed to grapple with mathematical language as explainers, justifiers, and reasoners. Jamie described explainers as a role in which:

Students are wrestling with a problem and trying to make clear their understanding of what the problem is asking, what prior knowledge is important for the situation, and what ideas they have for moving towards a solution. As they are working, they can explain to me or their classmates what they are doing (personal interview, 2018).

Jamie categorized explaining as a recall mechanism where they may explain to her or their peers their mathematical thinking. She wanted students as 'explainers' because she felt strongly that learning was about communicating their thoughts and ideas, and thus, students had to find voice in the classroom.

Another role Jamie wanted students to assume was that of justifiers. A justifier was a role assumed when a student had "a solution and they are using the math they have

done to prove their answer is mathematically sound”(personal interview, 2018). Jamie saw justification as a role necessary in answering “why” a solution or particular method was effective. Finally, Jamie wanted students to assume a role of reasoner. This was the only role that Jamie felt she could not assume as the teacher. For Jamie, a reasoner was a role happening behind the scenes and may not always be explicit. When asked to define a reasoner, Jamie stated:

Reasoning is happening throughout the process of working, explaining, justifying. That is something they have to do and own individually. However, I think listening to others explain and justify can support a student's reasoning, which is why collaboration plays such an important role in learning and understanding math (personal interview, 2018).

She explained that she could not assume a role of reasoner because as a teacher she “can explain math and justify a solution, but I cannot reason for my students, reasoning is almost more of a personal, internal student role”(personal interview, 2018). Thus, Jamie wanted her students to assume roles of explainer, justifier, and reasoner when engaging and grappling with mathematics in her classroom. Jamie saw reasoner as a role unique, individualistic, and almost internal to students. As reasoners, students were making sense of the mathematics and arriving at some sort of mathematical understanding, which she did not think she could provide for students.

I think I was providing justification and explaining, but I can't provide reasoning and understanding. But when they're doing the telling, they can. They can provide justification and explaining themselves as well as understanding; I can't provide that [understanding] for them.

Given the three roles of explainer, justifier, and reasoner that Jamie wanted her students to assume, Jamie viewed those roles as having direct implications on the roles she must assume.

Summary

Interviews with Jamie revealed her negotiated and relationally determined understanding of positioning. With a limited variety of roles to assume, Jamie first made sense of positions of teacher and student through herself. She saw herself and her positioning as the entry into this work. By focusing on herself, she believed she could become more deliberate and intentional in the positions she assumed. This granted her the opportunity to assume positions she believed more suitable for inquiry-based mathematics instruction. Roles of listener, questioner, observer, and facilitator were roles she saw herself assume in her previous lessons, but these were also the roles she wanted to work toward assuming more in the future. Given her understanding of roles as mutually exclusive, such that she and her students could not simultaneously assume the same role, she believed that she must first step out of her authoritative and sense-making role to encourage and support students in trying on these more agentic roles.

Jamie saw the work of positioning students starting with the teacher. Using a backward design approach, she thought of the skills and positions she wanted students to assume. Much of her work hinged on student dialogue and students being given the opportunities to make sense internally, with partners and small groups, and as a collective

class (not necessarily in that order). Given that student dialogue was important to her, she knew she must amplify student voice by quieting her own and learning to negotiate timing.

Summary of Findings

Storylines in Jamie's classroom changed across three dimensions. Some storylines remained consistent throughout her instruction. Those storylines included *teaching means telling*, *teaching means eliciting kids' thinking*, and *learning means listening*. Within those storylines Jamie most often assumed positions of *authority* and *questioner* and students were afforded and accepted limited positions of *followers*.

Other storylines dissipated as Jamie reconsidered her role and meaning of teaching and learning. *Teaching means equitable access*, *teaching means organization*, *learning means having a starting place*, and *learning means organization* were storylines no longer observable after three years of professional learning. One reason for this could be the melting of these storylines into the structures of classroom norms established in her classroom. Organization of work need not be made explicit because this was an engrained norm in this classroom space. Similarly, Jamie and her students worked collectively to engage in the mathematics such that equitable access and getting started no longer needed to be explicitly addressed during the lesson. While the storylines dissipated, the positions of authority and questioner for Jamie and follower and sense-maker for students persisted. Simply put, the storylines went away; available positions for teacher and student remained.

Finally, findings supported the emergence of storylines and positions. *Teaching means pressing kids to justify, teaching means facilitating discussions, teaching means making connections, and teaching means listening* were storylines that emerged over time and were not observed in spring 2015 lessons. Complementary meanings of learning math emerged as well including *learning means communicating, learning means asking questions, and learning means sense-making*. These meanings of teaching and learning pushed Jamie into a new position of *listener* and students into new positions of *authority* and *questioner*.

Findings from research question two suggest that by working with teachers in a reflective space and viewing classroom videos with a lens toward positioning, a teacher can dig deeply to consider the relational and negotiated nature of positioning with their students. As teacher and researcher work collaboratively, we may facilitate learning with practicing teachers to (1) recognize positioning, (2) consider the impact of reflexive positioning on others, specifically students in the room, and (3) be intentional in renegotiating storylines and what it means to teach and learn mathematics to re-position students as more influential and agentic mathematicians. More powerful positions for students allows us to push beyond mathematics as a gate-keeper (Martin, Gholson, & Leonard, 2010; Stinson, 2004) and support the development of mathematical identities through repeated positions of authority, power, and sense-making for students (Anderson, 2009; Suh, Theakston-Musselman, Herbel-Eisenmann, & Steele, 2013).

CHAPTER V

DISCUSSION

In this dissertation, my aim was to use Positioning Theory as an analytical and theoretical lens to identify and trace changes in prevalent storylines around teaching and learning and positions in a teacher's classroom as she engaged in sustained professional development around instructional practice. Additionally, I attempted to understand Jamie's perception of the relational and negotiated nature of positioning through video-stimulated recall interviews. Findings from this study showcase the meanings made of teaching and learning mathematics, the related positions assumed, and the nuanced ways those shifted over time. Storylines of teaching and learning mathematics shifted in three distinct ways throughout the project. Some storylines remained relatively consistent with limited change, other storylines and positions emerged, and some storylines were no longer evident in Jamie's practice. Six crosscutting positions were identified including *follower*, *mathematical authority*, *questioner*, *listener*, *facilitator*, and *sense-maker*.

These positions were cross-cutting in that any participant, teacher or student, may assume each position, but not all participants were given equal access. Additionally, while positions were available to each participant, some were more often assumed and afforded within certain storylines (i.e. *Teaching means telling* most often positioned Jamie as a mathematical *authority* and students as *followers*). Section one of Chapter 4 described storylines in relation to change throughout three years of professional development. In

section two, I discussed Jamie's perceived relationship between reflexive and interactive positioning as being fluid between participants, but also mutually exclusive, such that when she assumed a role, that role was no longer made available to students until she assumed another role.

In what follows, I first provide a brief discussion of findings organized by research question to situate them in the research literature. I discuss changes in storylines over time (research question one) and the ways *teaching means telling* changed in relation to timing and intended purpose. I also address instructional moves as a vehicle of entry for considering positioning and finally describe the benefits of a limited number of positions. Moving forward to research question two, I begin with Jamie, her learning throughout this process, and specifically highlight her reflection on her reflexive positioning and its impact on her classroom interactions. While I agree that teachers are the natural, more knowledgeable leader within a classroom, this is not a teacher's only role. Rather, there is a time and place to position oneself as authority, but an equally viable and meaningful opportunity to position oneself as a listener, and the negotiation and timing of that intentional reflexive positioning is remarkably challenging. Finally, I discuss the implications on practice and future research.

Research on Storylines

Research in the field has suggested that there are often two camps for mathematics instruction, and though they go by many names, direct and dialogic teaching is one way to describe these models. As discussed in chapter two, the work of everyday teaching likely falls neither in direct or dialogic categories solely. Rather, the

accumulation of approaches and the meanings made over time of teaching and learning are what shapes and defines student's mathematical dispositions and identities toward content. What is more, while the teaching practices may be similar or even the same in dialogic or direct instruction, the practices are implemented differently for different purposes in dialogic or direct instruction (Munter et al., 2015).

Direct vs. Dialogic

Findings from this study support the assumption that a teacher's instruction is neither direct nor dialogic in isolation, but rather that teaching is a blending of both over time, with similar pedagogical practices for different purposes. Storylines more closely aligned to direct instruction (i.e. *Teaching means telling, teaching means organization, teaching means eliciting kids' thinking, learning means listening, learning means organization, etc.*) were those that remained present in Jamie's practice throughout the study or were molded into classroom norms for engagement. Such storylines also afforded positions more closely aligned with direct instruction including *authority, follower, and questioner*. Others in the field have also noted the presence of authoritative positions of teachers in the classroom (Sheets, 2005; Wagner & Herbel-Eisenmann, 2014) and "telling" practices of teachers (Lobato, Clarke, & Ellis, 2005). Though not directly discussed in terms of positioning theory, student positions of follower and listener are similar to characterizations of students as passive recipients of knowledge in direct instruction classrooms. Nuanced shifts in the implementation of practices and the purpose for them were made evident in positions and storylines, and while examining Jamie's purpose for using practices was beyond the scope of this study, the approach

might be a productive way to investigate storylines and positions moving forward in future research.

While some findings of storylines and positions are consistent with literature around more direct models of mathematics instruction, other findings did not align as closely. Those storylines more aligned to dialogic instruction (i.e. *Teaching means pressing kids to justify, teaching means facilitating discussions, teaching means making connections, teaching means listening, learning means making connections, learning means sense-making*, etc.) were those that emerged in Jamie's classroom and were not present in much of the literature in the field. Such storylines afforded positions for Jamie less focused on authority and more centered on *facilitator, questioner, and listener*.

Taken together, such storylines and positions suggest a teaching and learning in mathematics involves listening, discussing, questioning, and building on others ideas.

Below, I move to discuss salient storylines previously documented in the literature and some potential reasons for the absence of others identified in this study.

Storylines of Equity

Many scholars have used positioning theory to highlight inequitable access for students from historically marginalized populations and have identified storylines about gender (Esmonde, 2011; Tholander & Aronsson, 2003), maturation (Suh et al., 2013; Thompson et al., 1994), and student ability (Herbel-Eisenmann et al., 2015; Wilson et al., 2017). My findings provided no observable instances of storylines around gender, age and grade, or maturation. Perhaps those storylines did not exist, but the methods and design of this study did not support me in uncovering such storylines if they were present.

To understand deeply issues of groups of students and their experiences in this mathematics class, one would need more lessons throughout the semester. Thus, I only had data for two classroom lessons (one fall and one spring) for one group of students. This structure limited my scope and issues of equity were not feasible given the time constraints. I wanted deeply to understand Jamie's progression and meanings made over time around teaching and learning mathematics. Thus, Jamie was the constant, not students. This focus on Jamie allowed me to speak more directly to her understood and communicated meanings of teaching and learning, as well as the assumed and afforded positions within storyline. In what follows, I move to discuss the evolution of storylines and positions.

Storyline and Position Evolution

Findings from research question one indicate there were shifts in storylines over time across three trajectories. Some storylines were sustained as Jamie engaged in professional development around her practice, other storylines dissipated, and others emerged. For this discussion, I focus broadly on three points in reference to storyline and position development in Jamie's classroom over time. First, one storyline in particular, *teaching means telling*, was categorized as a persistent storyline across all lessons. Here however, I discuss Jamie's adjustments in purpose and timing of this particular storyline. Next, I discuss instructional moves as a possible entry point for teachers to begin work with explicit and intentional positioning of self and others. Finally, I address the benefits of limited positions for teachers and students such that we can deeply consider implications of the relational nature of positioning.

Teaching means Telling

Teaching means telling was a storyline that commonly framed episodes during classroom interactions from spring 2015 to spring 2018. While this storyline remained consistent, the substance and timing of episodes did not. The purpose of Jamie's practice and when particular storylines were called forth were markedly different from initial lessons to final lessons. Below, I discuss shifts in purpose and timing in Jamie's classroom interactions.

Purpose. Lobato and colleagues (2005) pushed the field to recognize and legitimize telling as a sound pedagogical decision by considering 1) the form and function of telling, (2) the nature of telling as conceptual or procedural, and finally (3) telling in relation to other actions, rather than telling as an isolated act and decision made by the teacher. While some acts of telling are undesirable in mathematics classrooms as it lessens the cognitive load for students, if we consider the points above, we find that not all acts of telling are inherently "bad practice". While the form and function of Jamie's *teaching means telling* storyline is interesting, much of the form and function were captured in communication acts and positioning. What was more compelling about episodes following the *teaching means telling* storyline was the nature of conceptual or procedural telling and telling moves in relation to other actions. These two points were not as clearly captured in positioning triads.

While *teaching means telling* did not change as Jamie progressed in her practice, the *nature* and purpose of telling did change over the course of the study. The nature of Jamie's pedagogical telling initially focused on telling next steps, telling formulas, and

telling students how to approach tasks. Much of her telling minimized student voice, sense-making, and led students down a mathematically narrowed path (Lobato et al., 2005). Later lessons showcased telling moves more focused on pushing students to consider alternative strategies and methods, amplifying student voice and ideas, and orienting students to one another. The nature of telling became more conceptually focused instead of following procedural processes. Student became more comfortable during moments of uncertainty and silence became more acceptable as students were granted the time to sit and grapple with tough ideas rather than being “saved” through telling of next steps, which suggests the next shift in this storyline around issues of timing and when teaching means telling.

Timing. Though the nature of Jamie’s telling grew to be more conceptual in nature (Lobato et al., 2005), another notable shift in the *teaching means telling* storyline was timing. Initial episodes following this storyline in spring 2015 occurred shortly after students were given the math task for the day. The discomfort of not knowing how to begin was a new feeling for students in this classroom, due in part at least, to their educational labels of “advanced” and “gifted”. That discomfort and uncertainty was recognized by Jamie, and perhaps, Jamie felt unease in students struggle as well. Jamie’s attempt to alleviate the discomfort came by her almost immediately telling them how to begin and what steps they should do next to proceed. The immediacy of making a move to tell underscored Jamie’s power and closed mathematical exploration for kids. In episodes from later lessons in spring 2017 and spring 2018 specifically, Jamie was hesitant to tell. Jamie seemed to intentionally walk around the room and NOT engage

with groups when first given the task, and when she did approach a group, she would listen, wait, pause, pose questions, and may eventually tell, but telling was no longer her initial instructional move. Thus, while *teaching means telling* was a storyline that remained present throughout the three years of the study, the nature and timing of telling was markedly different and became more aligned with a more dialogic model for teaching and learning.

Instructional Moves

Within positioning theory, the constructs of positions, communication acts, and storylines are mutually determining and negotiated factors. However, in this work, I have often conceptualized communication acts as the “verb” or action as my entry into the work and the medium through which positioning is negotiated. For example, if a teacher stands at the front of the room and tells students to organize their thinking in a table of values and students accept and follow her lead, then I take the communication act of her stance, her physical location, and the words she is verbalizing as evidence of her authoritative position and a storyline centered on teaching means organization. The act and the utterance mark the episode of interaction and becomes the positioning theory construct through which I begin.

Findings from this study suggest that instructional moves, as verbal and non-verbal communication acts, are themselves positioning acts as well. Within the PD structure, specific instructional moves of *revoicing*, *probing*, *pressing*, *explaining*, and *orienting* were discussed as a means to support student investigations of mathematics, their voice, and their sense-making. What was not considered during the professional

learning, however, was the unintentional reflexive and interactive positioning that occurs through particular moves and series of those moves. As Jamie worked to incorporate moves from the PD into her practice, her reflexive positioning shifted. By considering a different purpose for an *explaining* move, utilizing it at a different time, or within a complex series of moves, Jamie re-positioned herself and redefined the meaning teaching and learning within that space. Her skillful uses of *orienting* moves supported students' mathematical authority, while simultaneously suspending her own. It seems as though work around instructional moves could provide an access point for teachers to begin thinking about their own positioning in the classroom while also considering the ways in which they want students to be positioned during the learning process. Thus, instructional moves may provide the window into power differentials in the classroom, issues of access and equity, and provide space for teachers to become more aware and intentional in positioning dynamics.

Fixed Positions

During professional development, our team worked deliberately to ensure that instructional moves discussed were limited such that teachers would have a manageable set of moves to pull from during rehearsals. Instead of a move in isolation, we focused (1) on the goal(s) of the move and (2) the coordination of several moves to elicit student thinking and make connections, rather than a move in isolation. The coordination and limited moves gave teachers the opportunity to dig deeply and practice a variety of moves. Findings from this study provided me with some of the same reflections as we had with the professional development. In my methodology, I made the decision to allow

positions to be fluid from teacher to student, such that either teacher or student may assume any position, but I also limited the number of positions to be assumed to six including, *follower, authority, sense-maker, listener, facilitator, and questioner*. By limiting the number of positions available in my data collection and analysis, Jamie and I were able to dive deeper into her own positions and their implications for students' positioning.

Jamie's Story

In research question two, I worked to make sense of Jamie's conceptions of positioning and its impact. During interviews with Jamie, two recurring themes were discussed that I want to describe in further detail below. First, Jamie's reflexive positioning of self and the ways she placed herself, either deliberately or unintentionally, in classroom interactions provided an entry for an initial dialogue around her practice. While watching recorded classroom videos, Jamie's first comments were her noticings and reflections of her own positioning, movement, and communication. While there are a variety of reasons for this, perhaps, self-positioning was easier to dissect and critically analyze. As interviews began, I prompted Jamie to "share what stood out and what she noticed", Jamie consistently began with her own movement, dialogue, and role in the classroom, conceptualized here as reflexive positioning. During many video-stimulated recall interviews, Jamie alluded to issues of timing, which is another point I address here. Jamie noted *when* she asked particular questions, *when* she assumed certain roles, and *when* she provided supports (or not) to students. Thus, the point in each lesson *when* roles were assumed was of great importance to Jamie. As a teacher who consistently

works to improve her practice, Jamie was quick to critique her choices and label them as good, bad, right, or wrong. However, while reflecting, we worked purposefully to remove “bad” and “good” qualifiers from particular roles and narratives, and instead, decided to focus on what we could learn moving forward. In what follows, I discuss reflexive positioning and timing and their possible impact on narratives of teaching and learning in Jamie’s classroom.

Reflexive Positioning

A position of power or authority, some may argue, is necessary of a teacher. Teachers are often a natural authority within a classroom space, asserting their power when determining rules, procedures, and the progression of content, to name a few. What is more, teachers’ language and non-verbal cues during interactions often set an authoritative tone that invites students to ‘follow the leader’. Inarguably, teacher positions of power and authority are necessary at times, and yet, there are many situations in which positions of mathematical authority and power can and should be afforded students. But how is the passing of power negotiated and when should it happen? Understanding the negotiation *with* and timing *of* interactive, powerful positioning of students is multi-layered, complex, and so deeply rooted in context that is no right or best way to achieve.

Positioning Theory makes clear that verbal and non-verbal communication acts position each participant in particular ways throughout interaction. Throughout three years of classroom interactions, the nature of discourse and communication acts within, followed Jamie’s lead. Further, students rarely contested her reflexive positioning or

their imposed interactive positions. And while episodes about the meaning of teaching and learning were most often initiated by Jamie, positions *are* negotiable, both participants mutually negotiate and accept/reject positions.

Davies and Hunt (1994) spoke to the ways that teachers often limit the positions available to students based on their preconceptions of what competence looks like. Through video-stimulated recall interviews and guided reflections, Jamie noted the impact her power, influence, and narrow assumptions of student competence had on storylines called forth and positions afforded to students. If awareness is the first step in reframing deficit discourse and storylines in classrooms, then once aware, a teacher can purposefully challenge and contest those more natural positions of power and authority and interactively afford more agentic positions of authority and sense-making to students.

Video-stimulated recall interviews provided a platform of awareness for Jamie. Over years of classroom lessons, Jamie reflected on her and her students' mathematical power in the classroom and quickly noticed that those positions of power rarely happened simultaneously. Rather, her power seemed to stifle kids' power and conversely, kids embodied her release of power. Reflecting an almost inverse relationship, Jamie became aware of her inquisitive and questioner positions during interactions and noted the impact on student positions. Teachers are natural mathematical authorities, and this is not going to change, nor should it. Perhaps instead, if we work together to leverage teachers' power to intentionally re-position themselves in classroom interactions, we can move students from underprivileged, mathematical positions to positions of greater power, showcased in their mathematical voice and ownership.

Timing

While knowledge and explicit awareness of Jamie's reflexive positioning was necessary and an entry point for us to discuss teacher and student positions collectively, more pivotal was an understanding of the negotiation between herself and students of *when* to assume particular positions and call forth storylines. Specifically, Jamie came to consider that she need not always assume a position of authority or consistently pose questions. Rather, she worked to better understand when particular positions were more/less appropriate within her classroom context.

During early video-stimulated recall interviews, Jamie commented on her authoritative positioning occurring during initial exchanges with a group or soon after students began digging into her task. Her earlier assumption of powerful roles not only set the tone for the remaining interaction, but also interactions to come. As we reviewed Jamie's spring 2017 and 2018 lessons however, she noted her reluctance to tell and explain the mathematics and instead, approached groups from a stance of inquiry; she asked questions about students' thinking, their approach, and where they were heading. Specifically, *teaching means telling* and *learning means listening* were referenced almost immediately during conversations, positioning teacher as *authority* and students as *followers*. Once those storylines and positions were enacted and assumed, Jamie noted the difficulty for herself and her students to step out of those roles.

With initial positions and storylines seemingly set the tone for interactions that followed, Jamie noticed later lessons (specifically spring 2017 and spring 2018) followed strikingly different scripts. Instead of Jamie first assuming a position of power, she

shifted to an inquirer and worked to consistently question students. This shift, from *authority* to *questioner* early in conversation and early in lessons worked to establish patterns for interactions in spring 2017 and 2018 that were markedly different from interactions in spring 2015 and 2016. In her retrospective observations, Jamie noted her shift in position pushed students toward more agentic and authoritative positions within interactions and persisted as lessons progressed. Thus, Jamie felt timing of positioning was something that must be considered in her work moving forward in developing her practice. Not only did she note the need for intentionality in assuming different, less authoritative positions, but she also eloquently captured and made note of timing—*when* she assumed those positions was of significant importance in reframing practice such that students saw themselves as mathematical contributors and sense-makers in the classroom space.

Two points were discussed here in reference to Jamie's perception of positioning. First, Jamie considered her own position first and used her positioning, dialogue, stance, and non-verbal cues to understand roles students were afforded or denied. Entry into positioning work started with an inward and critical lens on her own practice. Second, Jamie consistently noted timing of positioning. *When* she assumed an authority role was just as significant to her as *how* she assumed that role, as well as the implications of that role on students role choices.

Implications for Practice and Research

Above, I addressed discussion points for each research question and highlighted the complexities of positioning work. Moving forward, I draw on these findings and

discussion to address implications for practice and future research before the conclusion of this dissertation.

Practice

Findings from this study suggest two implications for improving the practice of teaching. First, a teacher's explicit attention to reflexive positioning is difficult work. However, this introspection undoubtedly has the potential to support student voice, autonomy, and agency in the classroom. Thus, supporting teachers in attending to their own positioning during classroom interactions was vital. Jamie worked backwards to first consider positions and meanings of learning mathematics she wanted students to embody, and then to consider the implications of learning and student positioning had on her teaching and teacher positioning. Second, and relatedly, the meaning of teaching and learning mathematics can be said to be different sides of the same coin. What a teacher believes to be important to learning mathematics has implications for teaching mathematics and vice versa. Careful consideration of what meanings about learning and teaching mathematics are being communicated is a necessary springboard to consider positioning of players within those meanings of teaching and learning.

Research

Findings from this study suggest a few points for future research I discuss below. First, future research using Positioning Theory would benefit from explicit attention to communication acts and specifically, non-verbal actions taken by participants. Similarly, research to catalog and trace possible connections between those non-verbal cues and

related positions. Second, Positioning Theory, I argue, has the potential to bridge the gap between issues of equity and pedagogy in professional development.

Communication Acts. The field has defined communication acts to be those verbal and non-verbal cues of participants in interaction. What has not been documented as clearly however, is what counts as an act, specifically, a non-verbal act. The field would benefit from explicit attention to cataloged, non-verbal communication acts. Positions, storylines, and even speech acts have been documented in the field, but what has not been captured are those non-verbal cues that suggest, support, and/or refute particular positions and storylines.

Positioning as a Bridge. As teacher educators, it is important to honor a teacher's practice, but also push towards more equitable practice and access for each student in mathematics. Positioning Theory, though a complex theory to untangle, I argue has the potential to bridge the gap between practice and equity. Attention to only instructional practice is not enough to ensure all students' access and success in mathematics. Similarly, attention only to issues of equity in mathematics may seem disjoint from everyday instructional practice. Rather, I suggest future research and work with pre-service and practicing teachers with an eye toward positioning has the potential to impact both teachers' practice and address inequities and power dynamics prevalent in classroom spaces. What is more, considering reflexive positioning seemed less intrusive and created safe space for Jamie to consider her and students' positioning to begin addressing issues of inequity, but also served as a medium for discussing instructional practice and moves to support high-quality mathematics learning for all.

Conclusion

The meanings made of teaching and learning mathematics in most secondary classrooms restrict access for many students and are unproductive when we think about developing positive mathematical identities in students. However, as teachers commit to their work on practice, over time and with opportunities for meaningful reflection, teachers can work to reshape and reframe unproductive meanings of teaching and learning. As meanings around teaching and learning mathematics shift in a classroom space, so too will the roles of teachers and students during interactions. As teachers work intentionally to remove themselves from authoritative and powerful, knowledge-holder roles into different roles, opportunities may be created for students to step in and assume those previously unattainable roles. Those sense-making positions for students supports students and their peers in viewing themselves as mathematical contributors and develops their mathematical identities moving forward.

The context in which this work happens is important. Though we know that typical instruction is a hybrid of both direct and dialogic approaches, there are commonalities between the two models and differences exist when we consider the intent and purpose in those practices. Teachers need work with core practices and developing an understanding and appreciation for high-quality mathematics instruction such that they can begin more intentionally incorporating more dialogic focused practices in their instruction. As classrooms become more entrenched in mathematical discourse, argumentation, reasoning, and justification that builds mathematical fluency, students

may experience growth and what was once considered stagnant mathematics achievement will be disrupted and more students experience success.

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APPENDIX A

VIDEO-STIMULATED RECALL INTERVIEW PROTOCOL

Introduction

- *Thank you for your willingness to participate in this study. I know you are busy and your time is valuable, so I truly appreciate you!*
- *I will audio record this interview so that I may focus on our conversation instead of writing notes during our time. This interview will take no longer than 1 hour.*
- *I want to discuss some of the roles you take on and afford to your students during classroom interactions.*

Interview #1—Storyline Development

{This interview aligns with RQ#1 and RQ#2. This interview will use member-checking to challenge/confirm prevalent SL's (RQ1) and their shift over time (RQ2)}

1. I want you to watch the following clips and tell me what stands out to you and what you notice.
2. What does teaching mathematics mean to you?
 - a. Using evidence from the videos, what did teaching mathematics mean for you 3 years ago?
 - b. What has it come to mean for you now?
 - c. Finish the following statement: As a mathematics teacher, my job is to...
(Watching clips again, but this time with an eye towards students and their interactions)
3. What does it mean to learn mathematics in your classroom?
 - a. Using evidence from the videos, what did it mean to learn mathematics 3 years ago?
 - b. What has it come to mean for you now?
 - c. Finish the following statement: As a mathematics learner, my job is to...
(Finally, reflecting on clips of teaching and learning to reflect on mathematical "smartness")
4. What would you say counts as being smart in mathematics?
 - a. What qualities might a "smart" math student possess?
 - b. Does "smart" in your classroom mean something different now than it did 3 years ago?
 - c. In an ideal mathematics classroom, what is evidence of smartness?

Introduction

- *Thank you for your willingness to participate in this study. I know you are busy and your time is valuable, so I truly appreciate you!*
- *I will audio record this interview so that I may focus on our conversation instead of writing notes during our time. This interview will take no longer than 1 hour.*
- *I want to discuss some of the roles you take on and afford to your students during classroom interactions.*

Interview #2—Positions and Communication Acts

{This interview aligns with RQ#1 and RQ#2. This interview will use member-checking to challenge/confirm prevalent positions (RQ1) and their shift over time (RQ2)}

1. I want you to watch the following clips and tell me what stands out to you and what you notice. Anything that you did not notice last time?
2. For this interview, we will focus on your role as the teacher and zoom in on your thoughts/words/actions.

(Watching clips again, but this time with an eye towards teacher and her spoken words and unspoken actions)

- a. What do you notice about your actions? Your language?
 - b. If asked to define or label your role in this interaction, what would you say?
 - c. Is there anything that surprises you about this interaction?
 - d. **You mentioned that as a teacher, you assumed XXX role, do you think this is a role that students could take on?**
3. Now, let's watch the clip again. This time, I want you to focus on the student(s) and their role in this interaction.

(Watching clips again, but this time with an eye towards students and their spoken words and unspoken actions)

- a. What do you notice about student actions? Student language?
- b. If asked to define or label students' role in this interaction, what would you say?
- c. Is there anything that surprises you about this interaction?
- d. **You mentioned that as a student, they took on roles of YYY; do you think this is a role that you could take on as a teacher?**

Introduction

- *Thank you for your willingness to participate in this study. I know you are busy and your time is valuable, so I truly appreciate you!*
- *I will audio record this interview so that I may focus on our conversation instead of writing notes during our time. This interview will take no longer than 1 hour.*
- *Last time, we talked about teacher and student roles separately, today, I want to take this time to consider both student and teacher roles in the classroom.*

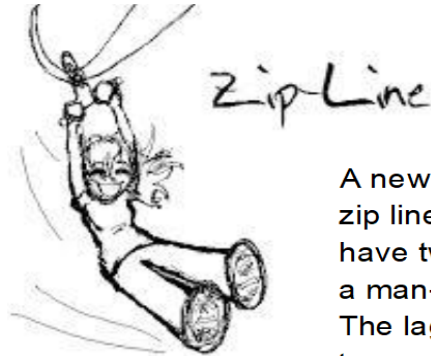
Interview #3—Positions of Teacher and Student

{This interview aligns with RQ#1 and RQ#2. This interview will use member-checking to challenge/confirm prevalent positions (RQ1) and their shift over time (RQ2)}

1. What are roles that you hope to take on as a teacher in your classroom? Why?
2. Do the roles you mentioned you want to assume as a teacher determine what roles students are granted?
3. Do you think that roles can be interchangeable?
4. Do you think students choose roles they want to take on?
5. Do you see any connections between your role and students' role? If not, why not? If so, why?

APPENDIX B

ZIP LINE (SPRING 2015)



A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600m wide. One tower will be 100m tall and the other will be 60m tall. There will be two zip lines, one from each tower, which riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of the zip line?

APPENDIX C

LAWNMOWER (SPRING 2016)

Lawnmower

Ali needs to save \$700 by the end of the summer for senior year expenses, so she takes a job with a local lawn service. The lawn service has had trouble with teenagers quitting the job after a few days, so they devised an interesting way of paying them for summer work. First, the service only allows teenage employees to mow two lawns per day. On the first day of employment, they pay \$2 per yard. On the second day, they will pay \$3 per yard. On the third day, they will pay \$4 per yard, and so on. Ali takes the job, but she is wondering how long it will take her to earn \$700. How much money has Ali made after n days of employment with the lawn service? How many days must she work to make \$700?

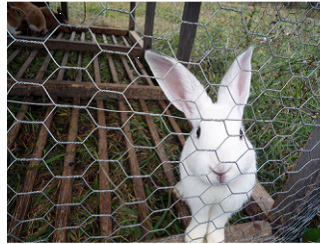
APPENDIX D

RABBIT RUN (SPRING 2017)

SECONDARY MATH II // MODULE 1
QUADRATIC FUNCTIONS—1.4

1.4 Rabbit Run

A Solidify Understanding Task



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<https://flickr/p/7yxyNv>

Misha has a new rabbit that she named "Wascal". She wants to build Wascal a pen so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen.

1. If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?
2. If Misha wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.

APPENDIX E

TAKING SIDES/INEQUALITIES (SPRING 2018)

24

SECONDARY MATH I // MODULE 4
EQUATIONS AND INEQUALITIES–4.6

4.6 Taking Sides

A Practice Understanding Task

Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently they had a discussion that went something like this:

Joaquin: The problem says that “6 less than a number is greater than 4.” I think that we should just follow the words and write: $6 - ! > 4$.

Serena: I don’t think that works because if n is 20 and you do 6 less than that you get $20 - 6 = 14$. I think we should write $! - 6 > 4$.

Joaquin: Oh, you’re right. Then it makes sense that the solution will be $! > 10$, which means we can choose any number greater than 10.



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The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide how to answer their questions, decide who is right, and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $! \geq -7$.
Joaquin thinks the graph should have an open dot -7.
Serena thinks the graph should have a closed dot at -7.
Explain who is correct and why.
2. Joaquin and Serena are looking at the problem $3! + 1 > 0$.
Serena says that the inequality is always true because multiplying a number by three and then adding one to it makes the number greater than zero.
Is she right? Explain why or why not.

APPENDIX F

CODEBOOK FOR INSTRUCTIONAL MOVES

Probing	Definition	Asking an “ information seeking ” question based on information students have verbalized or recorded about their understanding of the task, mathematical representation of the task, mathematical work, or mathematical statements.
	Examples	<i>What did you guys come up with?</i> <i>Where did you get these numbers from?</i> <i>Show me how you set this up?</i> <i>Does this match up with what is labeled on your triangle?</i> <i>what are you guys going to do to help solve this problem?</i> <i>What do we have to do before we solve for x?</i> <i>Why did you cross multiply?</i>
Pressing	Definition	Asking a question or making a statement that encourages students to explain or justify their reasoning beyond their initial explanations, to think more deeply about a mathematical idea, or extend their thinking to a new idea related to their understanding of the task, mathematical representation of the task, mathematical work, mathematical statements, or other students’ contributions.
	Examples	<i>Can you use the same idea, or do you have to use something different?</i> <i>Try setting it up a different way and see if you get the same number or a different number.</i> <i>Can we verify that this uses the least amount of zip line wire?</i> <i>Can you find some more solutions to see if that is the best solution or not?</i> <i>Can you find some math to back up what you are saying?</i> <i>If we think about this as an absolute value function, how is that going to help us figure out the location of the island?</i> <i>Is there a way we can show algebraically what is happening in the table?</i> <i>How could we take this and write a rule?</i> <i>Is there a way to prove mathematically what you just said?</i> <i>How could you prove or disprove what she is saying?</i> <i>How do you know this rectangle you created is the biggest area?</i>

Orienting	Definition	Asking a question or making a statement that encourages students to hear, use, or connect a student's or class idea or questions to their own idea related to their understanding of the task, mathematical representation of the task, mathematical work, mathematical statements, or other students' contributions.
	Examples	<p><i>How about Carla, does she have the same picture as you?</i></p> <p><i>So, talk to each other about why you chose Pythagorean Theorem.</i></p> <p><i>Caleb take your idea and apply it to her picture.</i></p> <p><i>Okay you have two ideas, she said set up to cross multiply and you said Pythagorean theorem...</i></p> <p><i>Turn and talk to your groups about how you would solve this problem.</i></p> <p><i>Each of you compare your numbers with each other.</i></p> <p><i>Jacob, as she is drawing, can you tell us what she is putting up there and what it represents?</i></p> <p><i>Do you mind showing that work you just talked about on the side of your paper, so you can see where it can from, so they can see it and you can explain it to the rest of your group?</i></p> <p><i>Kamin, can you share what you are working on with the rest of your group?</i></p> <p><i>Jalen make sure he understands where your numbers are coming from.</i></p>
Explaining	Definition	Making a statement to explicitly clarify to students an aspect related to the task, mathematical representation of the task, mathematical work, mathematical statements, or other students' contributions.
	Examples	<p><i>Minimize, it means the least amount of wire is going to be used.</i></p> <p><i>That is if you are dividing in half.</i></p> <p><i>Break this up into 2 pieces x and $600-x$.</i></p> <p><i>This is a right triangle, and this is a right triangle.</i></p> <p><i>Equal means congruent or the same.</i></p> <p><i>If you do it on one side, then you have to do it on the other</i></p> <p><i>Go back and read the problem again.</i></p> <p><i>Include that in your picture.</i></p> <p><i>We are trying to minimize the length of the wire and we need these distances.</i></p> <p><i>The only thing that will vary is the island location.</i></p> <p><i>The towers are set. That is the height.</i></p> <p><i>The shape of the wire is not the function.</i></p>

Revoicing	Definition	Restating a prior students' prior contribution by repeating or rephrasing statements related to the task, mathematical representation of the task, students' mathematical work or thinking, or students' mathematical statements.
	Examples	<i>These statements will be in direct response to a students' statement and will thus be a repeat or rephrase of what they said related to the task, mathematical representation of the task, students' mathematical work or thinking, or students' mathematical statements.</i>

APPENDIX G

RIVAL EXPLANATIONS OF FINDINGS

TYPE OF RIVAL	DESCRIPTION OR EXAMPLES
Craft Rivals:	
1. The Null Hypothesis	Storylines and positions observed only happened on these particular instances when Jamie was being observed and are not typical for her classroom culture/climate.
2. Threats to Validity	Lack of student interviews limited the varied interpretation of the storylines and positions discussed.
3. Investigator Bias	As a past colleague of Jamie's, I interpreted findings using a unique lens that other researchers may not utilize. Additionally, it is possible that Jamie performed in particular ways while I was in her classroom.
Real-World Rivals:	
4. Direct Rival	Evidenced storylines and positions are not the storylines of her particular classroom; rather, they are institutional and the product of years of schooling.
5. Commingled Rival	Task implementation and structure accounted for the storylines and positions evidenced in Jamie's classroom.
6. Implementation Rival	During implementation, Jamie simply mimicked PD facilitators and peers she has observed; this suggests storylines and positions were not unique to her classroom, but rather the result of collective work on pedagogical practice.
7. Rival Theory	Storylines and positions are not what explained the actions and norms in the classroom, rather the teacher's beliefs about teaching, beliefs of students and their capabilities, or even her teacher vision may account for evidenced storylines and positions in this classroom.
8. Super Rival	Evidenced storylines and positions are the result of the broader <i>school</i> culture, not her classroom culture.

Adapted from Yin (2013).

APPENDIX H

POSITIONS CODEBOOK

Positions	Examples
Mathematical Authority <i>A participant in an interaction that possesses mathematical knowledge relevant and necessary to successful completion of the task at hand</i>	<p>(5:5) J1: but I wasn't going to give it to you, I wanted you to find it. S: I wanted you to give it to me! J1: I'm going to give it to you now if you'll listen.</p> <p>(5:9) J1: Alright there's the formula (circles it on the board).</p> <p>(8:18) J1: What else? S: They both go to the – J1: (Whole Class) Yeah, they both go – there's an island in the middle, and you've got the two towers on the side, and the ziplines come down to the middle. Yes, okay? A lot of you are drawing a picture, that's good. I think if you draw the picture, you'll be able to see really easily what you need to do.</p> <p>(8:30) S: Okay, well – here's my question....is this right? J1: Well, what have you figured out?</p> <p>(8:57) S: Isn't there an exact equation for this? J1: There is. S: How do you figure out what it is? J1: I'm not going to tell you. If you can figure it out, then you can use it.</p> <p>(5:13) J1: There is your 300 point and what you found. (5:14) J1: You guys were doing, ya'll were making this table. You just didn't know the function for it and you got it all the way down. (6:6) J1: That's what you guys are figuring out.</p>
Facilitator	(7:9) J1: So Colby, you did a good job. Colby

<p><i>A participant in an interaction asking questions, calling on participants, and organizing information in meaningful ways</i></p>	<p>Doss, tell me what you guys found for your dimensions and area and what your reasoning was behind that.</p> <p>C: So, what we did was the shed could be anything we wanted it to be. So if we took out one of the sides from the square, one of them was 18, so then we decided to distribute it between all the other sides so $18/3 = 6$ so then we added 6 feet to all the other sides which would be 24 feet each.</p> <p>(7:15) J1: Okay, so they are getting 486 when they multiply that together, Alice.</p> <p>S: Cause I did that too and I got that.</p> <p>J1: Which is still bigger than the original 324 square feet pen, but not as big as the 24 x 24. Okay? So did anybody find anything else?</p>
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<p>Follower <i>A participant in an interaction who takes notes or creates a public record of mathematical thinking or ideas</i></p>	<p>5:3 J1: okay. Ya'll did a really good job. Alright. Listen. So, must of you figured out that you have this tower that is 60 and this tower that is 100 (drawing diagram on the board). This is your lagoon, which was a total of?</p> <p>5:7 J1: add together. So that would be $Y = \sqrt{60}$ squared, well what is 60 squared? S's: 3600 J1: (continues to write on the board) $+ x$ squared $+ \sqrt{10,000} +$ S's: ugh. 3600</p>
<p><i>A participant in an interaction who primarily follows another participant's lead without a full understanding of what or why a particular approach is appropriate</i></p>	<p>(5:16) J1: So in your table, you literally just scroll down and let me just change my table set real quick. Gonna change it to 1. Okay. It didn't work. S13: It did</p> <p>(8:52) S: We don't know. S: In a chart, but – J1: Okay, so who can take charge of creating that chart? S: I can. J1: Okay. S: Take charge of not understanding why we need a chart. S: This is hard.</p> <p>(8:55) S: I'm thinking about a graph, but I'm not sure. S: A chart? J1: Okay. S: I don't know how that could work though. J1: Okay, so who can be in charge of making that? S: I can.</p>

<p><i>A participant in an interaction who provides information requested by another participant—requiring limited thought or reasoning</i></p>	<p>(6:2) S: It's not straight across. J1: Exactly. What's it probably doing? (makes gestures with arms) S: sloping</p> <p>(8:65) J1: So, I would put those two and then the final outcome. And then those two and the final outcome. S: That's what I was thinking. S: How would we do the 60 and the 3? J1: I would make that your category because everything will time – this is the one that goes with the 60 and this is the one that goes with the 100. So, I would make that like your column headings. 60,100, and then total. S: Because we used to get – J1: So, then for your first triangle, the 60 had a base of 300.</p>
<p>Questioner <i>A participant in an interaction that asks questions (either probing or pressing) to other participants.</i></p>	<p>(11:4) J1: [Group 3] What's the length equal to? So, if you put that there, instead of that, could you do anything with that?</p>
<p>Sense-maker <i>A participant in an interaction who mathematically reasons and makes sense.</i></p>	<p>5:15 J1: Oh, it keeps getting smaller, we didn't get specific enough. (hits key on calculator to decrease the value). 621.42, 621.41, it keeps getting smaller. S15: it keeps going to like 225</p> <p>6:1 J1: There's some things that physically have to do</p>

	<p>with the problem and for the sake of mathematics, we are going to ignore. Like can somebody think of some of the things about a zip line that might make it more difficult to do this, that we can ignore?</p> <p>S: It's not straight across.</p> <p>8:14</p> <p>S: So, you need at least 206 meters. Yeah.</p> <p>J1: Why?</p> <p>S: Because if you put the towers one meter from the lagoon, then –</p> <p>J1: We're going to assume that the tower's right on the edge. So, they're 600 meters apart.</p> <p>S: So, you need 600 meters.</p> <p>8:20</p> <p>J1: And that would be how much you would need if it was dead center, but remember, you're trying to minimize the length of the zipline.</p> <p>S: Yeah.</p> <p>8:27</p> <p>J1: And how is that going to be the same thing because this one's taller?</p> <p>S: I don't know. Does it have to be the same length or –</p> <p>J1: No. Alright, so you figured out this is this, right?</p> <p>8:32</p> <p>S: Either way it's the same distance across.</p> <p>S: Yeah, either way.</p> <p>J1: Are you sure?</p> <p>S: Yes.</p> <p>S: No, because if you use the – if you use – this type of a – less steep slope around to work with. So, if you put that over there, then that'd be less line that putting this in the center, right?</p> <p>J1: See what happens.</p> <p>8:46</p> <p>J1: Yes, you do. You just don't want to do it.</p>
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	<p>S: I have no clue, like I still don't know. Honestly, I shouldn't even try.</p> <p>J1: Okay. So, you guys started by saying this is 300 –</p> <p>S: Yeah.</p> <p>J1: And this is 300. So, you figured out the distance of these two added together –</p> <p>S: We got 600 –</p> <p>J1: You got 622, right?</p> <p>S: Yeah.</p> <p>12:32</p> <p>S: The numbers. Like if you have a certain distance – She's like, "Why doesn't it work?" and I'm like, "It just doesn't." I'm gonna try 50, and then I'm gonna try a hundred and see.</p> <p>12:33</p> <p>J1: Alright, so these are – so, 280 and 300, you've still got 20 meters between those that you could play with. Well, you said you wanted to go between 280 and 300, right? You wanted to go between 280 and 300, right?</p> <p>12:37</p> <p>J1: Okay. So now, it's gotta be between those two, right? You've still got 100 meters between those two. And then, look, here, you went to the 50 point right here, so it's still in the same neighborhood.</p> <p>12:40</p> <p>J1: Is that – that's not even possible. That means that the wire would be attached to the zip line tower. That would be very painful if someone tried to do a zip line like that.</p> <p>S: Can it be one meter?</p> <p>J1: Well, that was 667, which is more than these three that you've already found.</p> <p>12:50</p> <p>J1 (Group 1): Alright at 220, you were at .96, and then, these are all higher than .96.</p>
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	<p>So, instead of – so when you go below the 220, do you see all these numbers are bigger? So, quit going below 220; y'all need to go what?</p> <p>S: Higher.</p> <p>J1: Yeah. Because these numbers are getting bigger.</p>
<p>Listener <i>A participant in an interaction who</i></p>	<p>(11:7) S: We're looking – we're trying to solve – we're looking for a quadratic function, so we can find the greatest factor, whatever it'd be. Because right now, we know it'd be concave down, because they're looking up. I can't like, we're looking for the possibilities of – since we know it would be length times width – are we looking for area?</p> <p>S: What's the possible area, yeah.</p> <p>S: Yeah, so right now, thinking about substituting zero in, but Casey said we should substitute zero in, so we can find the line intercept of the ratio.</p> <p>J1: So, what does your X stand for in your equation?</p> <p>S: Our space, personally?</p> <p>J1: So, remember, your Y intercept is where X is – so if the width was zero, what would the area be?</p> <p>S: Zero?</p> <p>J1: So, what's your Y intercept gonna be?</p> <p>S: Zero</p> <p>S: Zero.</p> <p>J1: Does that help some?</p> <p>S: Yes, it does.</p>